Advances in Averaged Switch Modeling and Simulation

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Outline

1. **Introduction: converter modeling approaches and objectives**
2. **Averaged switch modeling of PWM converters operating in the continuous conduction mode (CCM)**
   - Basics of averaged switch modeling
   - Switch network steady-state and small-signal models
   - Using averaged-switch model to predict converter steady-state characteristics and small-signal dynamics in CCM
   - PSpice implementation of the averaged switch model
   - Application examples: small-signal dynamics, conduction losses and efficiency of a Sepic converter
   - Averaged switch modeling exercise: include switching losses

Outline
continued

3. **Averaged switch modeling of PWM converters operating in discontinuous conduction mode (DCM)**
   - Averaged switch model in DCM
   - Switch network steady-state and small-signal models in DCM
   - Using averaged-switch model to predict converter steady-state characteristics and small-signal dynamics in DCM
   - Combined CCM/DCM averaged switch model
   - PSpice implementation of combined CCM/DCM models
   - Application examples:

     Large-signal transient response of a SEPIC

     Flyback converter small-signal frequency responses in CCM and DCM

Outline
continued

4. **Averaged modeling of PWM converters with current-programmed mode (CPM) control**
    - Averaged switch model in CCM and DCM
    - Steady-state and AC models in CCM and DCM
    - Large-signal averaged CCM/DCM model for CPM controller
    - PSpice implementation of the CPM controller model
    - Application example: buck converter with CPM controller
5. **Single-phase low-harmonic rectifiers**
    - The ideal rectifier
    - Averaged models of rectifiers
    - Application examples:

        DCM boost rectifier

        SEPIC rectifier with nonlinear-carrier control
6. **Summary**
7. **Bibliography**

On-Line Materials

- ***http://ece-www.colorado.edu/~pwrelect/publications***

  seminar slides, collection of simulation examples, library of PSpice models used in the examples, and many other CoPEC publications and presentation materials
- ***http://ece-www.colorado.edu/~pwrelect/*** is the CoPEC home page
- ***http://ece-www.colorado.edu/~pwrelect/book/bookdir.html***

  is the home page for the Textbook: R.W.Erickson, *Fundamentals of Power Electronics*
- **Power Electronics courses at the University of Colorado:**
  - *Power Electronics 1:* ***http://ece-www.colorado.edu/~ecen5797***
  - *Power Electronics 2:* ***http:// ece-www.colorado.edu/~ecen5807***
  - *Power Electronics Lab:* ***http:// ece-www.colorado.edu/~ecen4517***
- All simulation examples completed using free PSpice evaluation version available from: ***http://www.orcad.com***

1. Introduction

Engineering design based on converter modeling:

- Predict converter system behavior, validate models by experiments
- Use the model to predict performance under worst-case conditions
- Improve design until worst-case behavior meets specifications
  (or until reliability and production yield are acceptably high)

Models:

- Circuit models that yield design-oriented, analytical results
- Models for computer simulation

Results of interest:

- Steady-state characteristics
- Component stresses, losses, efficiency
- Large and small-signal dynamic responses

Seminar Objectives

- Describe basic averaged switch modeling approach
- Develop averaged models for
  - Converters in continuous conduction mode (CCM)
  - Converters in discontinuous conduction mode (DCM)
  - Converters with Current-Programmed Mode (CPM) controller
  - Single-phase power-factor correctors
- Summarize analytical results for steady-state and dynamic responses
- Demonstrate PSpice implementations of averaged-switch models and controllers
- Present application examples
  - Large-signal transient responses and small-signal dynamics of DC-DC converters and single-phase power-factor correctors

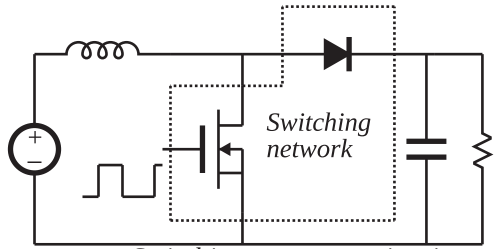
Averaged switch modeling

- Switch network is replaced by averaged circuit model. Switching harmonics are removed, and low-frequency components of waveforms are modeled in a simple way.
- A very general approach to modeling converter losses, efficiency, and dynamics.
- Yields an intuitive understanding of converter behavior in CCM, DCM, current-programmed mode, etc.
- Applicable to all types of converters: dc-dc converters, as well as dc-ac inverters, ac-dc low-harmonic rectifiers, ac-ac matrix converters.
- Well-suited to simulation
- Well developed and understood technique, easily taught to students.
- Main reference for the material in this seminar:

  R.W.Erickson, *Fundamentals of Power Electronics*, Chapman and Hall, 1997.

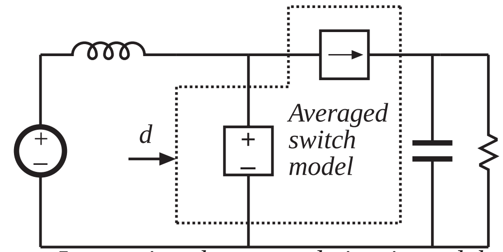
Bibliography has a large collection of other selected references

Averaged switch modeling



Switching converter circuit

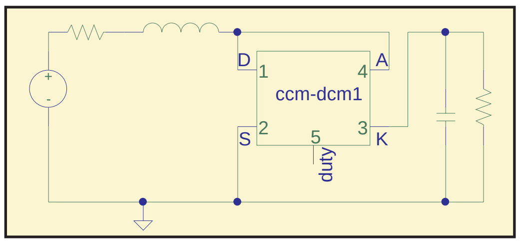
averaging



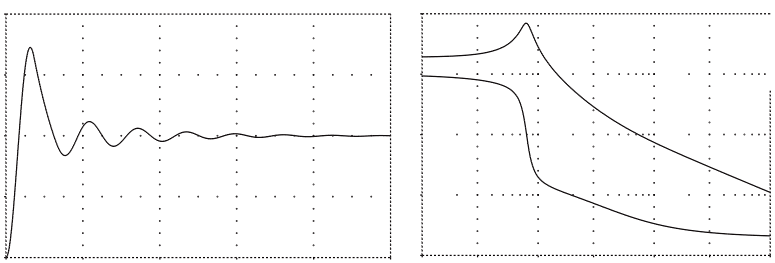
Large-signal averaged circuit model

simulation
model

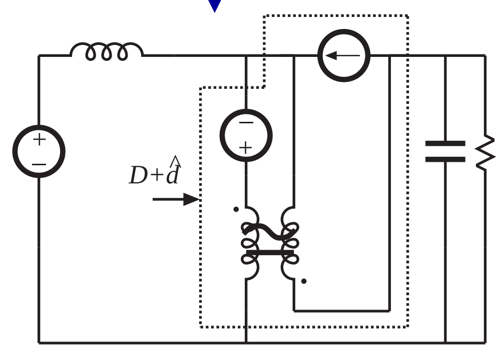
linearization



Model implementation for simulation



DC, AC and Transient simulation



DC and small-signal averaged circuit model

$$G_c(s) = G_{co} \frac{1 - s/w_s}{1 + (1/Q)s/w_o + (s/w_o)^2}$$

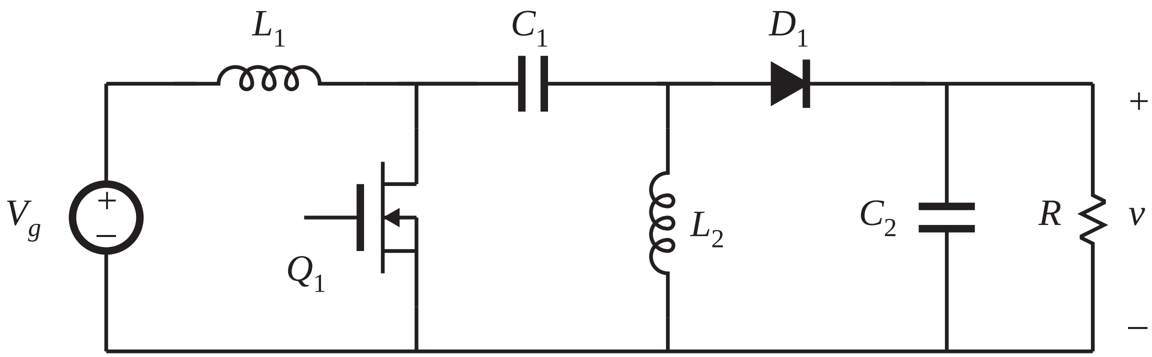
Analytical results:
steady-state characteristics
and small-signal dynamics

2. Averaged switch modeling of PWM converters operating in the continuous conduction mode

- **Basics of averaged switch modeling**
- **Switch network steady-state and small-signal models**
- **Using averaged-switch model to predict converter steady-state characteristics and small-signal dynamics in CCM**
- **PSpice implementation of averaged switch models**
  - ideal switches (ccm1)
  - switches with conduction losses (ccm2)
  - switches in converters with isolation transformer (ccm3)
  - switch with conduction losses in converters with (possibly) isolation transformer (ccm4)
- **Application example:**
  - SEPIC small-signal frequency response, conduction losses and efficiency
- **Averaged switch modeling exercise: include switching losses**

Averaged switch modeling
Basic approach

Given a PWM converter operating in continuous conduction mode:

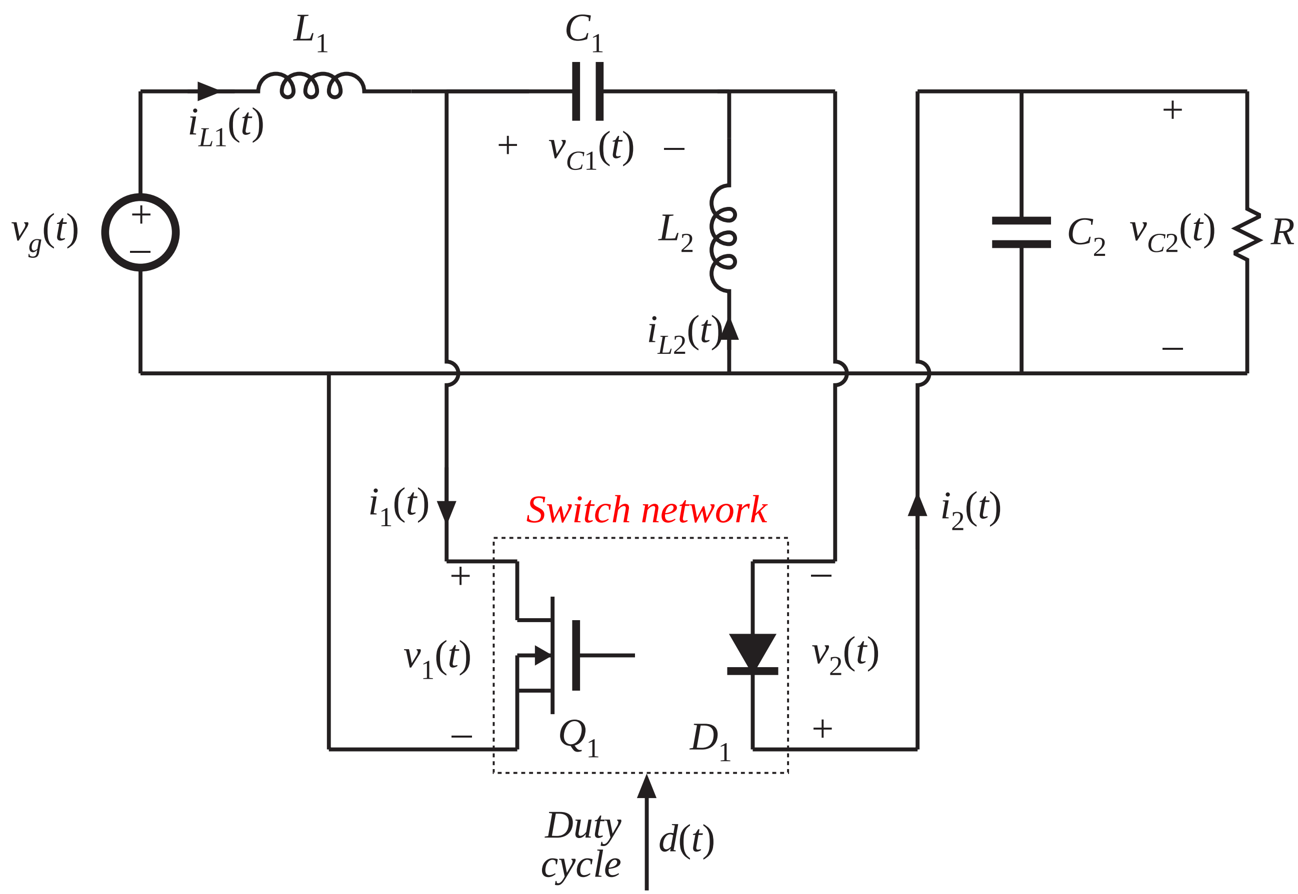


SEPIC example

Separate the switching elements from the remainder of the converter...

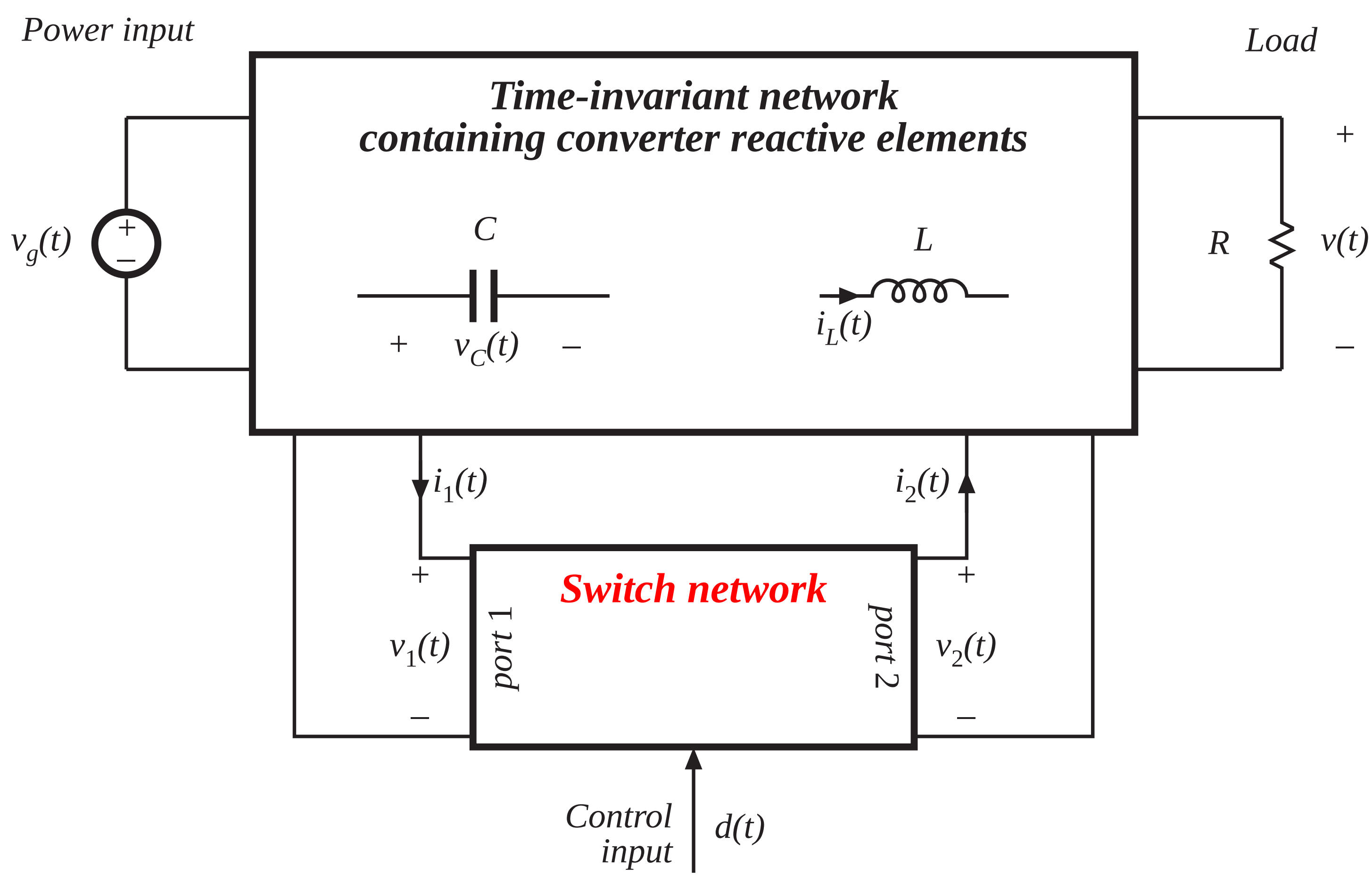
Definition of switch network, SEPIC example

- Define a switch network, containing all of the converter switching elements.
- The remainder of the converter is linear and time-invariant.
- The terminal voltages and currents of the switch network can be arbitrarily defined.



Switching converter system

with switch network explicitly defined



Discussion

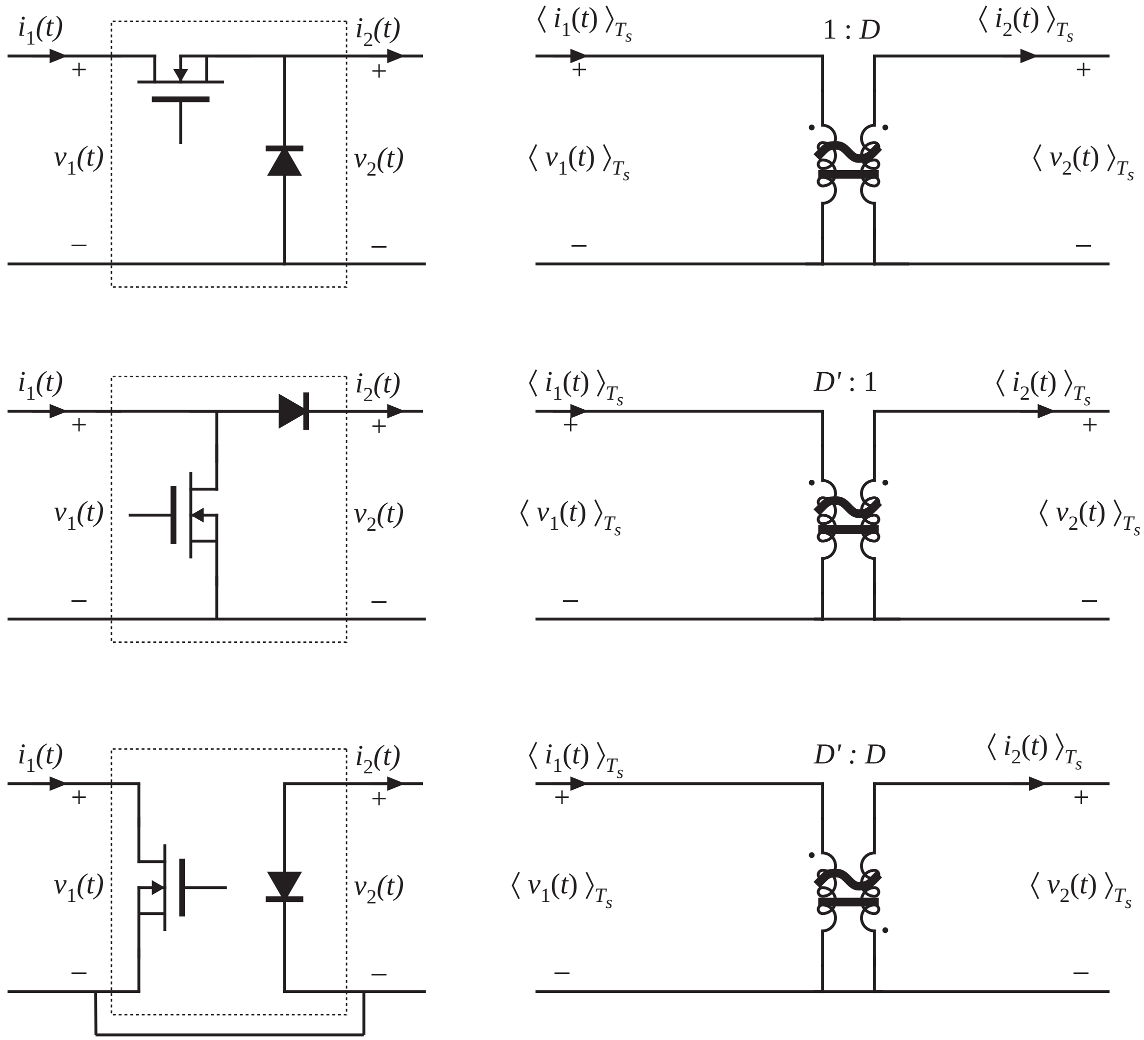
- The number of ports in the switch network is less than or equal to the number of SPST switches in the converter
- Simple dc-dc case, in which converter contains two SPST switches: switch network contains two ports

  The switch network terminal waveforms are then the port voltages and currents: $v_1(t)$, $i_1(t)$, $v_2(t)$, and $i_2(t)$.

  Two of these waveforms can be taken as independent inputs to the switch network; the remaining two waveforms are then viewed as dependent outputs of the switch network.

  Switch network also includes control input $d(t)$
- Definition of the switch network terminal quantities is not unique. Different definitions lead equivalent results having different forms

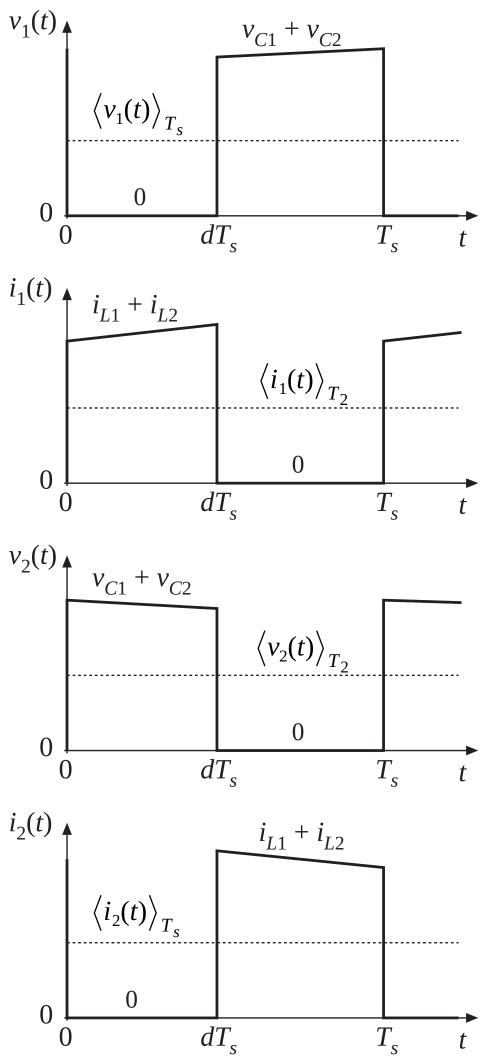
Several ways to define the PWM switch network, and the corresponding CCM models

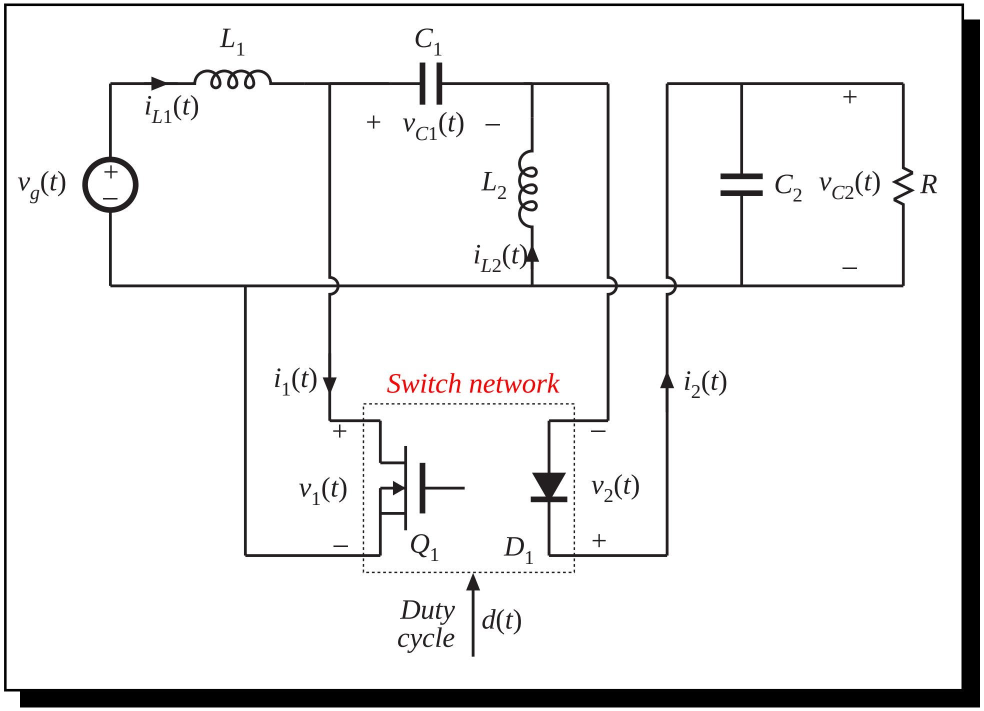


A few points regarding averaged switch modeling

- The switch network can be defined arbitrarily, as long as
  - its terminal voltages and currents are independent, and
  - the switch network contains no reactive elements.
- It is ***not*** necessary that some of the switch network terminal quantities
  - coincide with inductor currents or capacitor voltages of the converter, or
  - be nonpulsating.
- The object is simply to write the averaged equations of the switch network; i.e., to express the average values of half of the switch network terminal waveforms as functions of
  - the average values of the remaining switch network terminal waveforms, and
  - the control input.

Terminal waveforms of the switch network

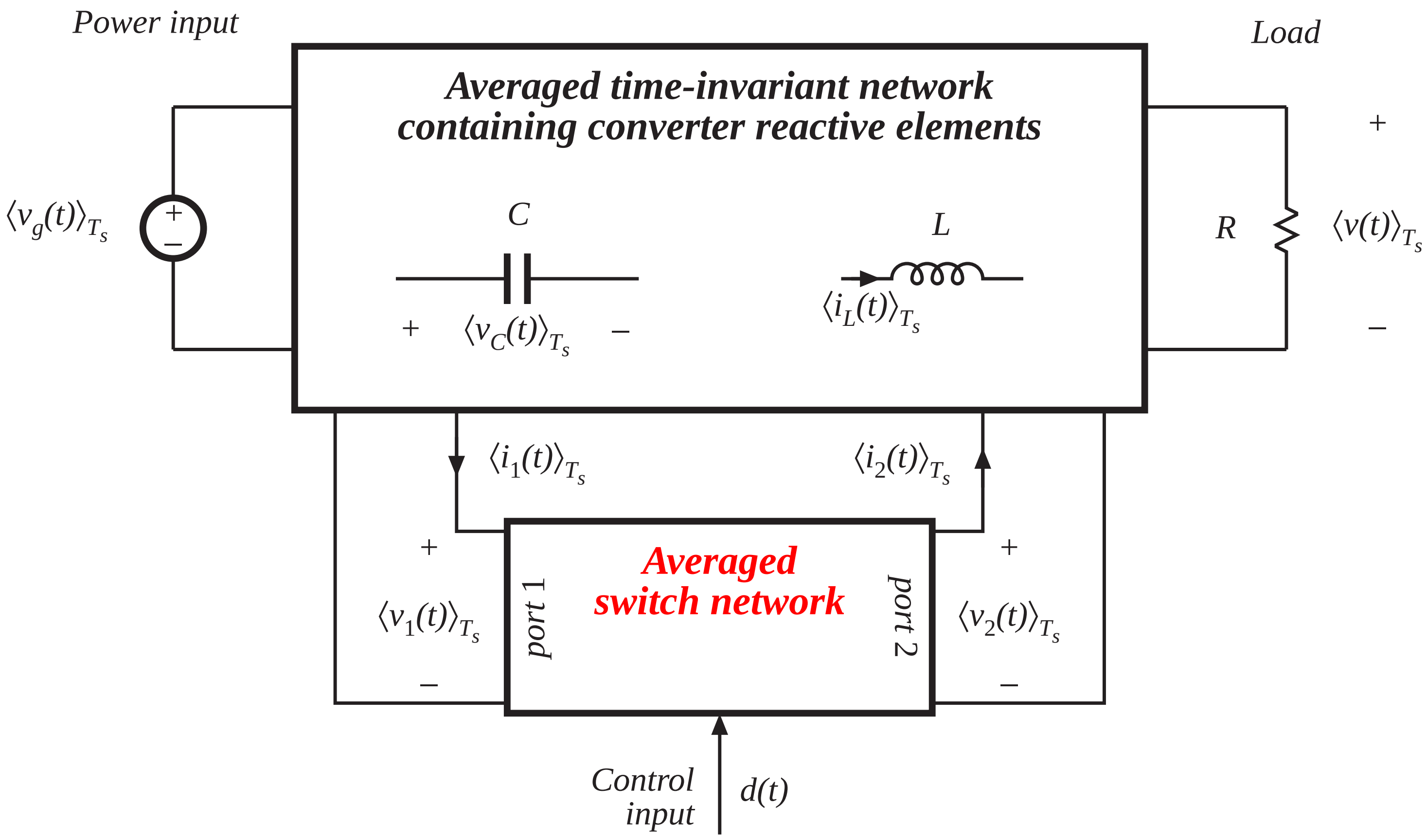




The averaging step

$$\left\langle x(t) \right\rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} x(t)\,dt$$

Now average all waveforms over one switching period:



The averaging step

The basic assumption is made that the natural time constants of the converter are much longer than the switching period, so that the converter contains low-pass filtering of the switching harmonics:

One may average the waveforms over an interval that is short compared to the system natural time constants, without significantly altering the system response.

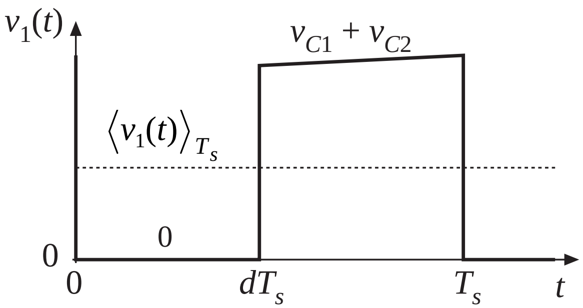
In particular, averaging over the switching period T_s removes the switching harmonics, while preserving the low-frequency components of the waveforms.

This step removes the small but mathematically-complicated switching harmonics, leading to a relatively simple and tractable converter model.

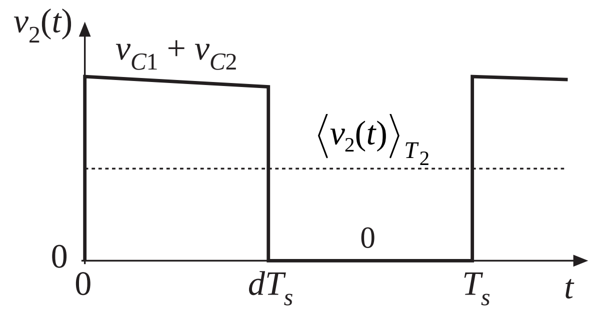
In practice, the only work needed for this step is to average the switch dependent waveforms.

Averaged terminal equations of the switch network

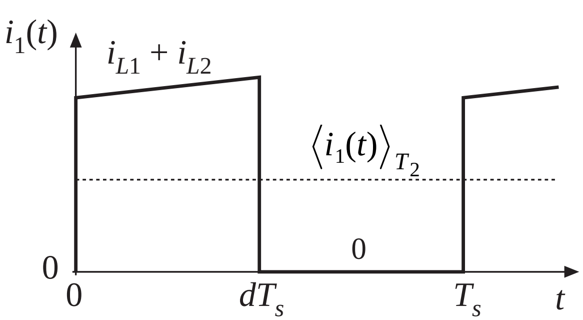
(small switching ripple is neglected)



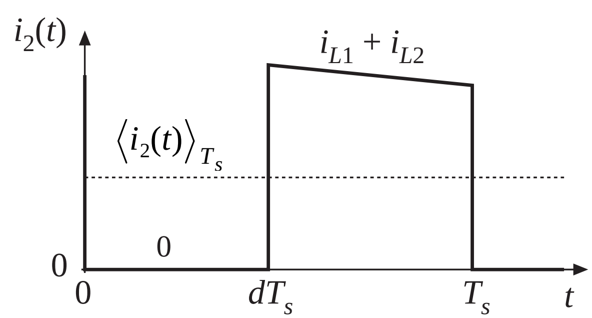
$$\left\langle v_1(t) \right\rangle_{T_s} = d'(t) \left(\left\langle v_{C1}(t) \right\rangle_{T_s} + \left\langle v_{C2}(t) \right\rangle_{T_s} \right)$$



$$\left\langle v_2(t) \right\rangle_{T_s} = d(t) \left(\left\langle v_{C1}(t) \right\rangle_{T_s} + \left\langle v_{C2}(t) \right\rangle_{T_s} \right)$$



$$\left\langle i_1(t) \right\rangle_{T_s} = d(t) \left(\left\langle i_{L1}(t) \right\rangle_{T_s} + \left\langle i_{L2}(t) \right\rangle_{T_s} \right)$$



$$\left\langle i_2(t) \right\rangle_{T_s} = d'(t) \left(\left\langle i_{L1}(t) \right\rangle_{T_s} + \left\langle i_{L2}(t) \right\rangle_{T_s} \right)$$

Derivation of switch network equations

(Algebra steps)

We can write

$$\left\langle i_{L1}(t) \right\rangle_{T_s} + \left\langle i_{L2}(t) \right\rangle_{T_s} = \frac{\left\langle i_1(t) \right\rangle_{T_s}}{d(t)}$$

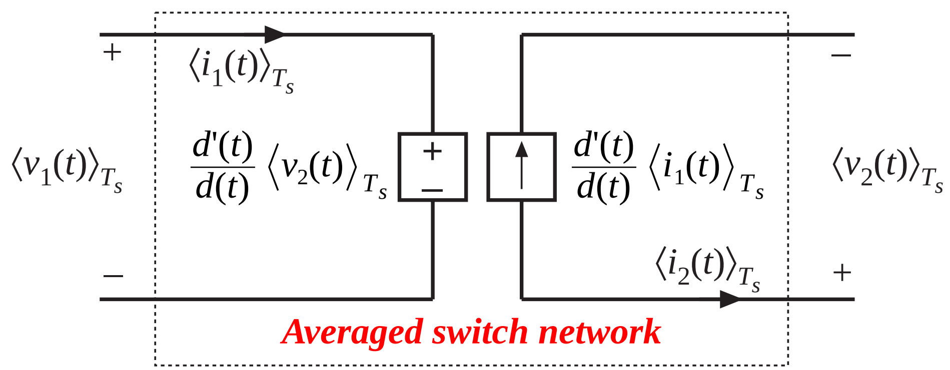
$$\left\langle v_{C1}(t) \right\rangle_{T_s} + \left\langle v_{C2}(t) \right\rangle_{T_s} = \frac{\left\langle v_2(t) \right\rangle_{T_s}}{d(t)}$$

Hence

$$\left\langle v_1(t) \right\rangle_{T_s} = \frac{d'(t)}{d(t)} \left\langle v_2(t) \right\rangle_{T_s}$$

$$\left\langle i_2(t) \right\rangle_{T_s} = \frac{d'(t)}{d(t)} \left\langle i_1(t) \right\rangle_{T_s}$$

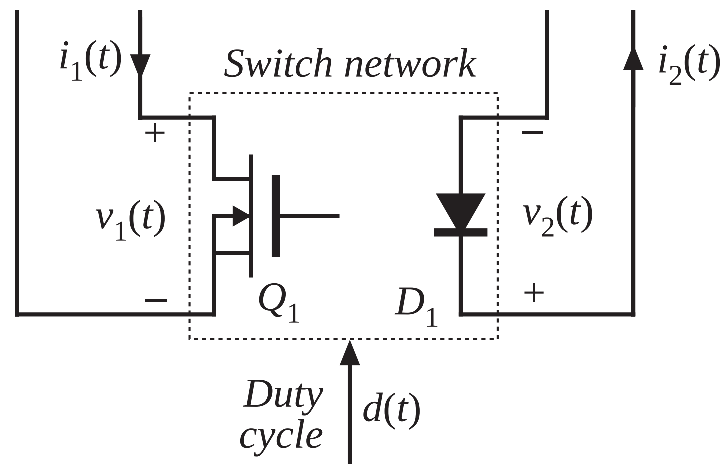
Result



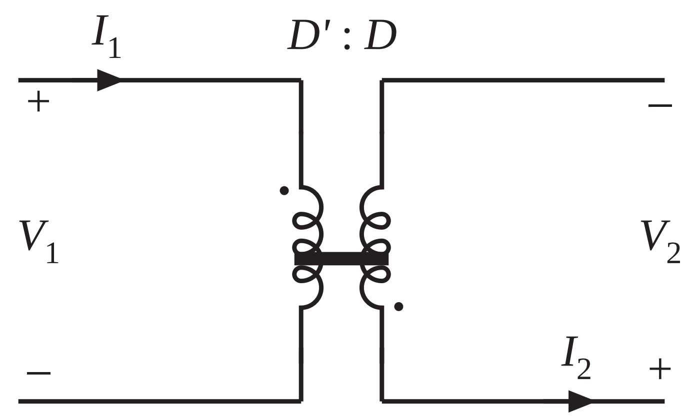
Modeling the switch network via averaged dependent sources

Steady-state switch model: Dc transformer model

Original switch network



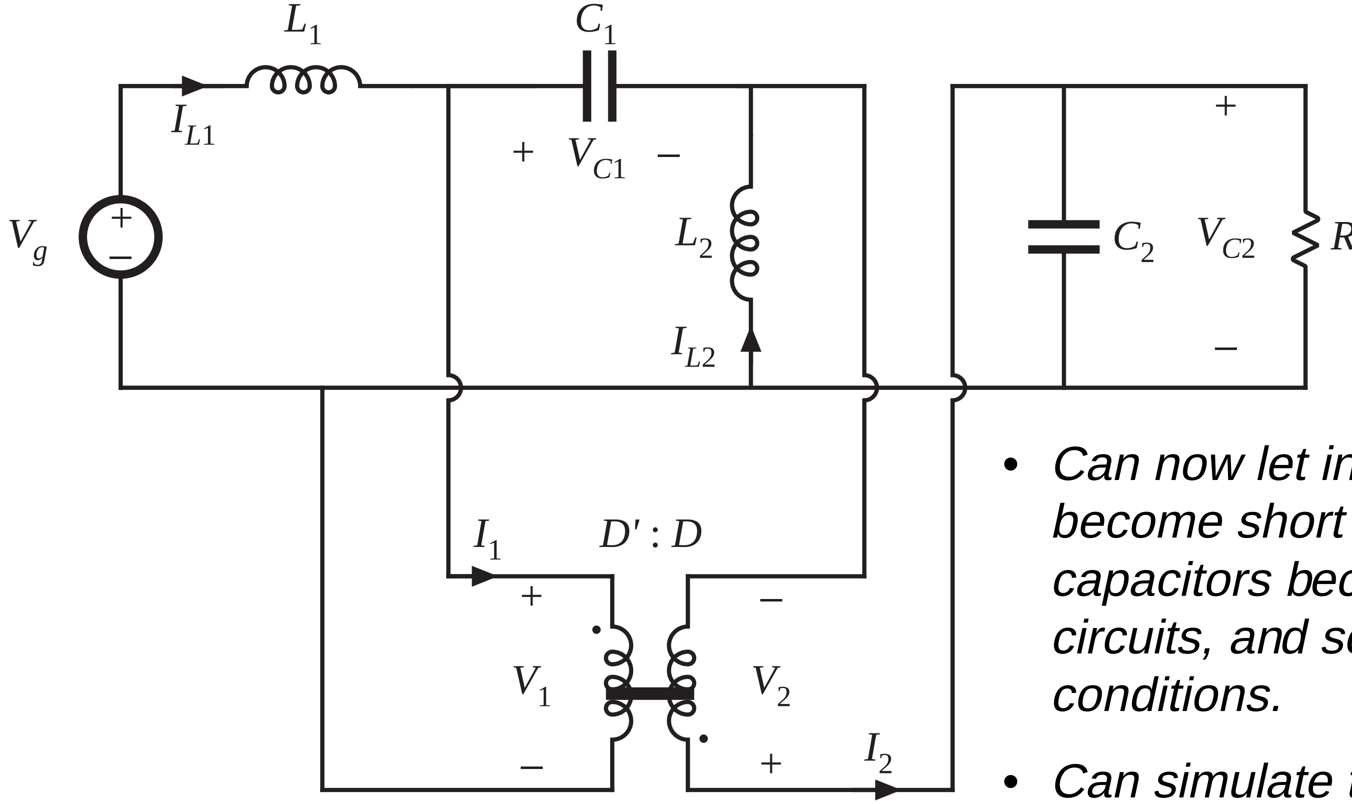
Averaged steady-state model:
"DC transformer"



- Correctly represents the relationships between the dc and low-frequency components of the terminal waveforms of the switch network

Steady-state CCM SEPIC model

Replace switch network with dc transformer model



- *Can now let inductors become short circuits, capacitors become open circuits, and solve for dc conditions.*
- *Can simulate this model using PSPICE, to find transient waveforms*

Modeling converter dynamics:
Small-signal linearization of model

Perturb and linearize the switch network averaged waveforms about a quiescent operating point. Let:

$$d(t) = D + \hat{d}(t)$$

$$\left\langle v_1(t) \right\rangle_{T_s} = V_1 + \hat{v}_1(t)$$

$$\left\langle i_1(t) \right\rangle_{T_s} = I_1 + \hat{i}_1(t)$$

$$\left\langle v_2(t) \right\rangle_{T_s} = V_2 + \hat{v}_2(t)$$

$$\left\langle i_2(t) \right\rangle_{T_s} = I_2 + \hat{i}_2(t)$$

Voltage equation becomes

$$\left(D + \hat{d}\right)\left(V_1 + \hat{v}_1\right) = \left(D' - \hat{d}\right)\left(V_2 + \hat{v}_2\right)$$

Eliminate nonlinear terms and solve for v_1 terms:

$$\left(V_1 + \hat{v}_1\right) = \frac{D'}{D}\left(V_2 + \hat{v}_2\right) - \hat{d}\left(\frac{V_1 + V_2}{D}\right)$$

$$= \frac{D'}{D}\left(V_2 + \hat{v}_2\right) - \hat{d}\left(\frac{V_1}{DD'}\right)$$

Linearization, continued

Current equation becomes

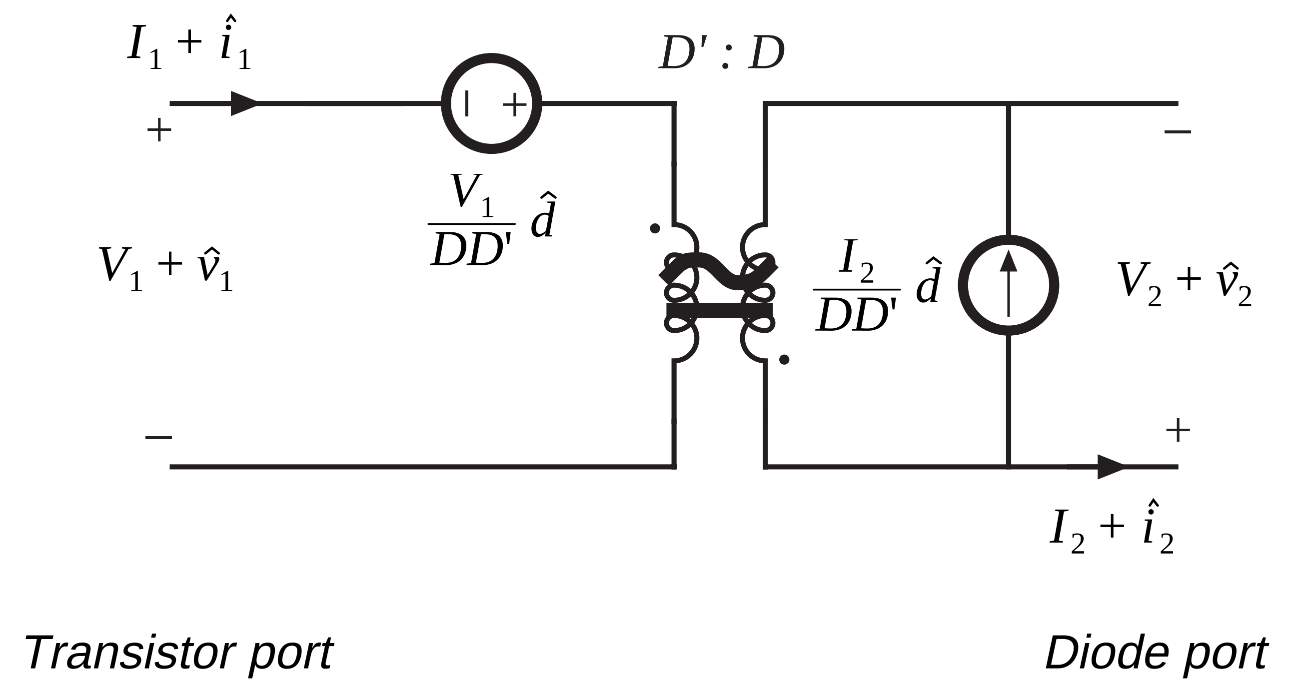
$$\left(D+\hat{d}\right)\left(I_2+\hat{i}_2\right)=\left(D'-\hat{d}\right)\left(I_1+\hat{i}_1\right)$$

Eliminate nonlinear terms
and solve for i_2 terms:

$$\begin{aligned}\left(I_2+\hat{i}_2\right)&=\frac{D'}{D}\left(I_1+\hat{i}_1\right)-\hat{d}\left(\frac{I_1+I_2}{D}\right)\\&=\frac{D'}{D}\left(I_1+\hat{i}_1\right)-\hat{d}\left(\frac{I_2}{DD'}\right)\end{aligned}$$

Switch network:
Small-signal ac model

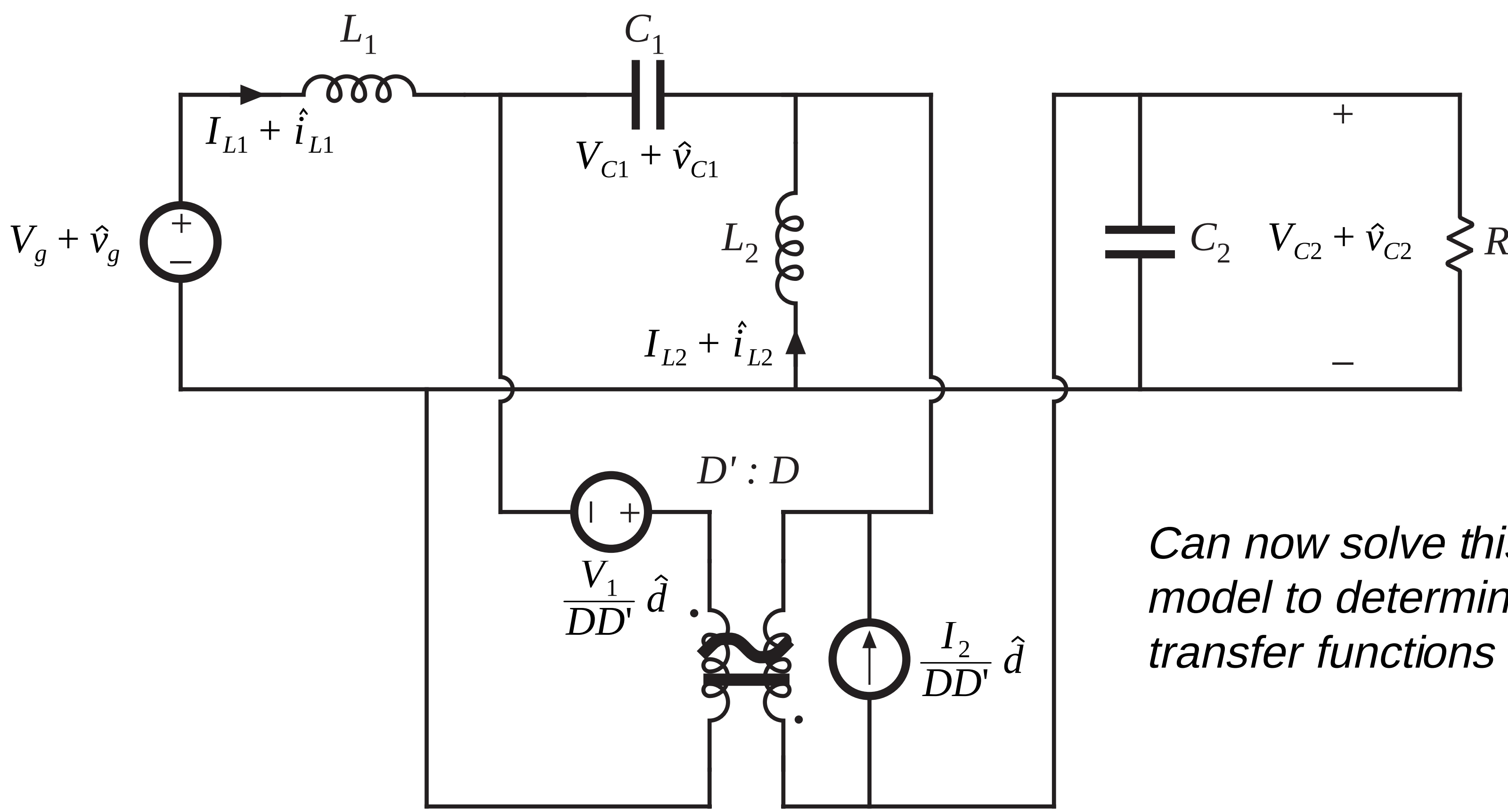
Reconstruct an equivalent circuit that corresponds to these small-signal equations:



A general small-signal ac model for the PWM switch network operating in CCM.

Small-signal ac model of the CCM SEPIC

Replace switch network with small-signal ac model:



Can now solve this model to determine ac transfer functions

Small-signal models of several basic switch networks

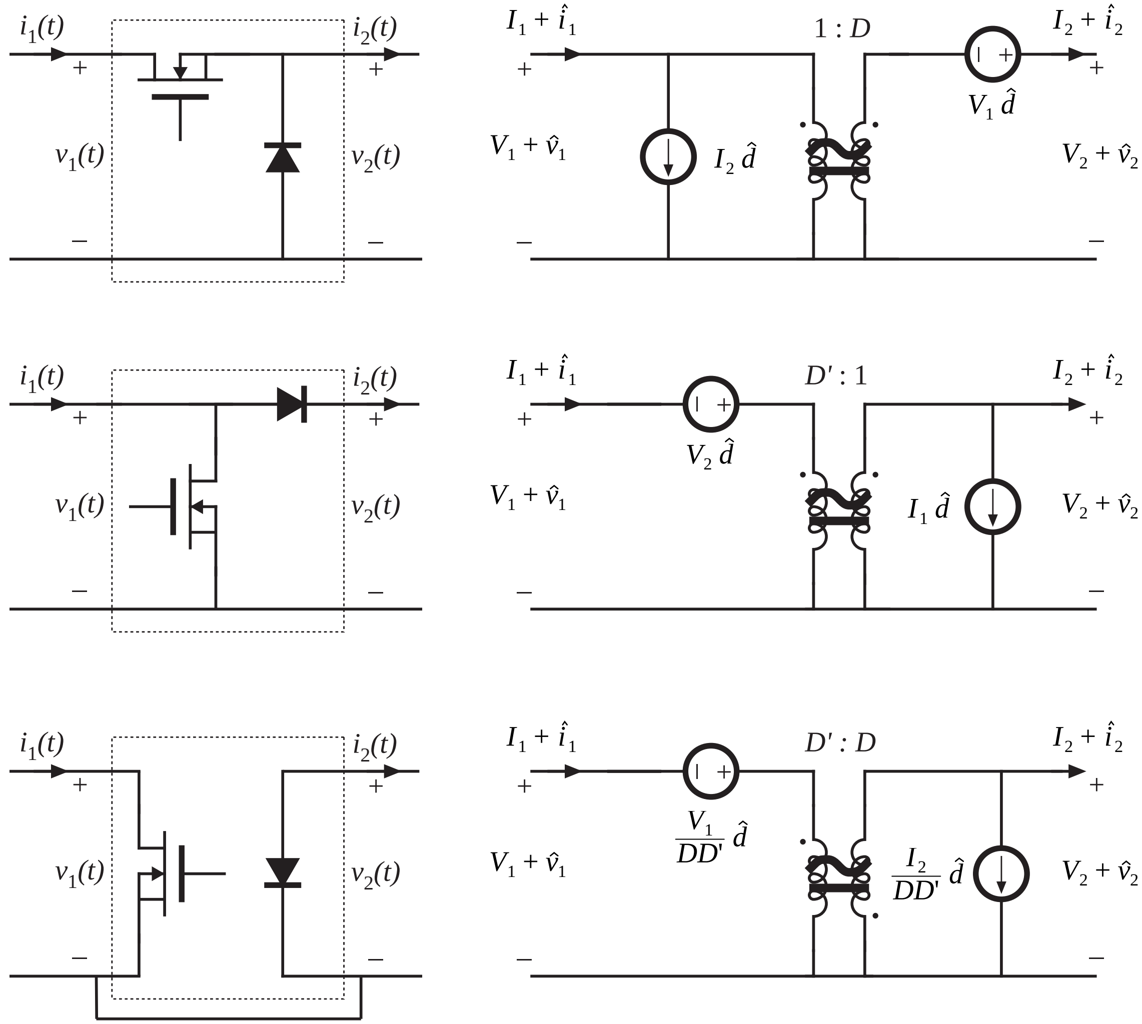


Table of results

Transfer functions of the basic buck, boost, and buck-boost converters

Control-to-output and line-to-output transfer functions $G_{vd}(s)$ and $G_{vg}(s)$

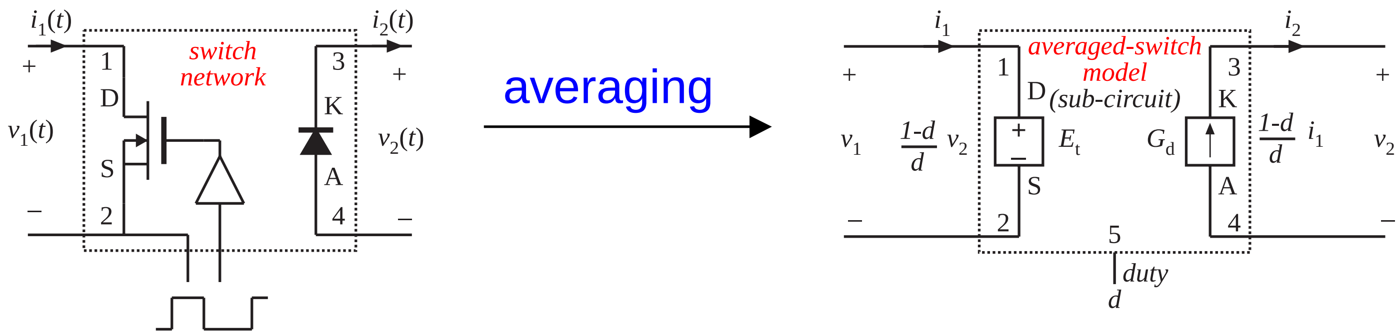
Converter	G_{g0}	G_{d0}	ω_0	Q	ω_z
buck	D	$\frac{V}{D}$	$\frac{1}{\sqrt{LC}}$	$R\sqrt{\frac{C}{L}}$	∞
boost	$\frac{1}{D'}$	$\frac{V}{D'}$	$\frac{D'}{\sqrt{LC}}$	$D'R\sqrt{\frac{C}{L}}$	$\frac{D'^2R}{L}$
buck-boost	$-\frac{D}{D'}$	$\frac{V}{D\,D'^2}$	$\frac{D'}{\sqrt{LC}}$	$D'R\sqrt{\frac{C}{L}}$	$\frac{D'^2R}{D\,L}$

where the transfer functions are written in the standard forms

$$G_{vd}(s) = G_{d0}\frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

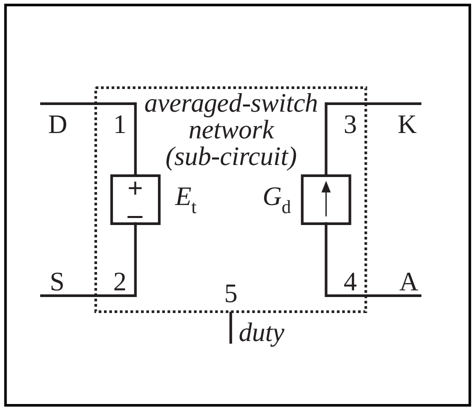
$$G_{vg}(s) = G_{g0}\frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

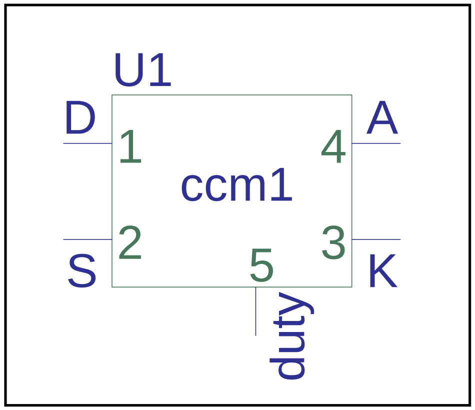
PSpice implementation of the basic CCM averaged switch model (ccm1)



- Controlled voltage source E_t replaces the transistor, controlled current source G_d replaces the diode
- Duty ratio *d* is input to the subcircuit
- Large-signal, nonlinear model suitable for DC, AC or Transient simulation
- The *same* model can be applied in *any* two-switch PWM converter (the transistor and the diode need not have a common node)
- Limitations: ideal switches, CCM only, valid for two-switch converters without isolation transformer

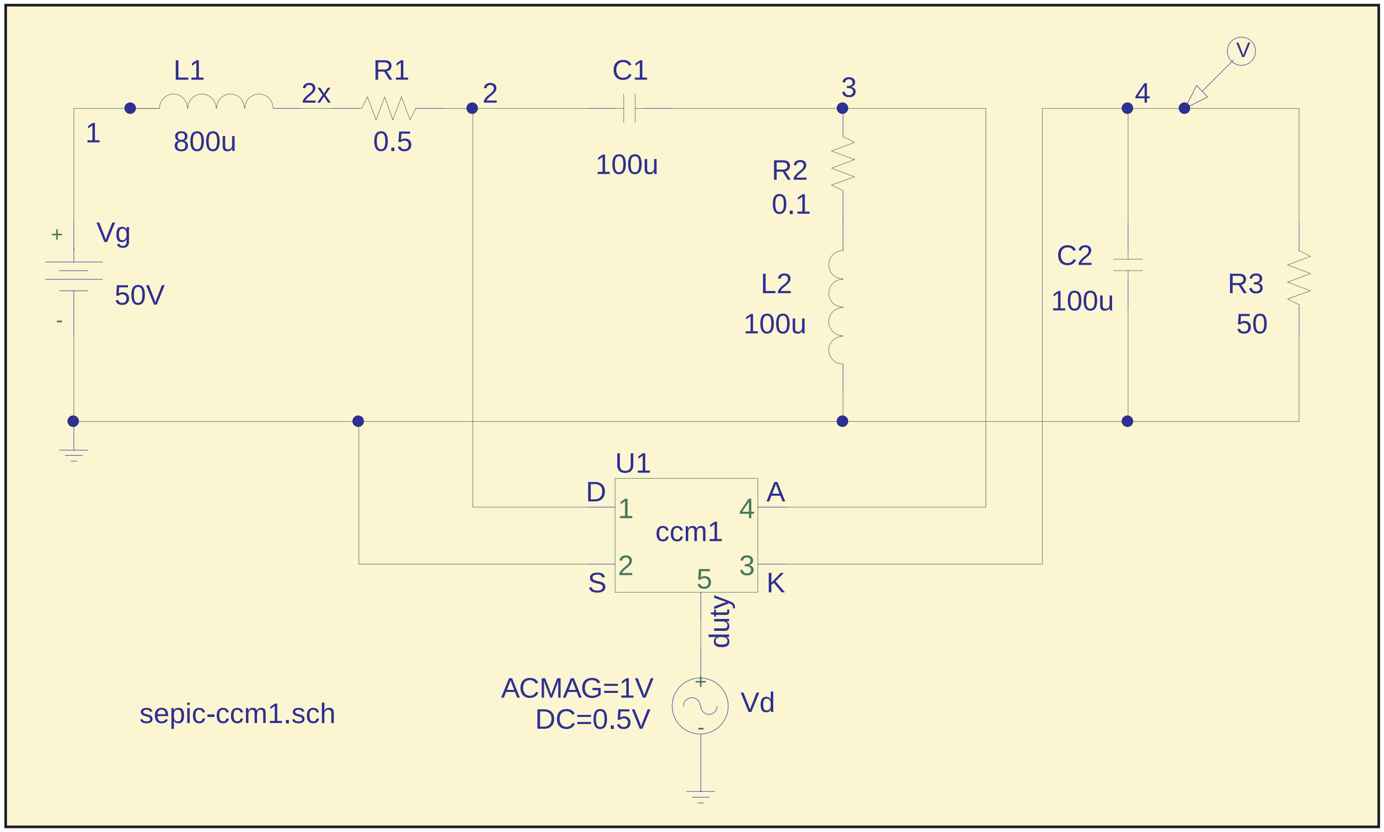
CCM Averaged-Switch Model
PSpice Implementation: ccm1





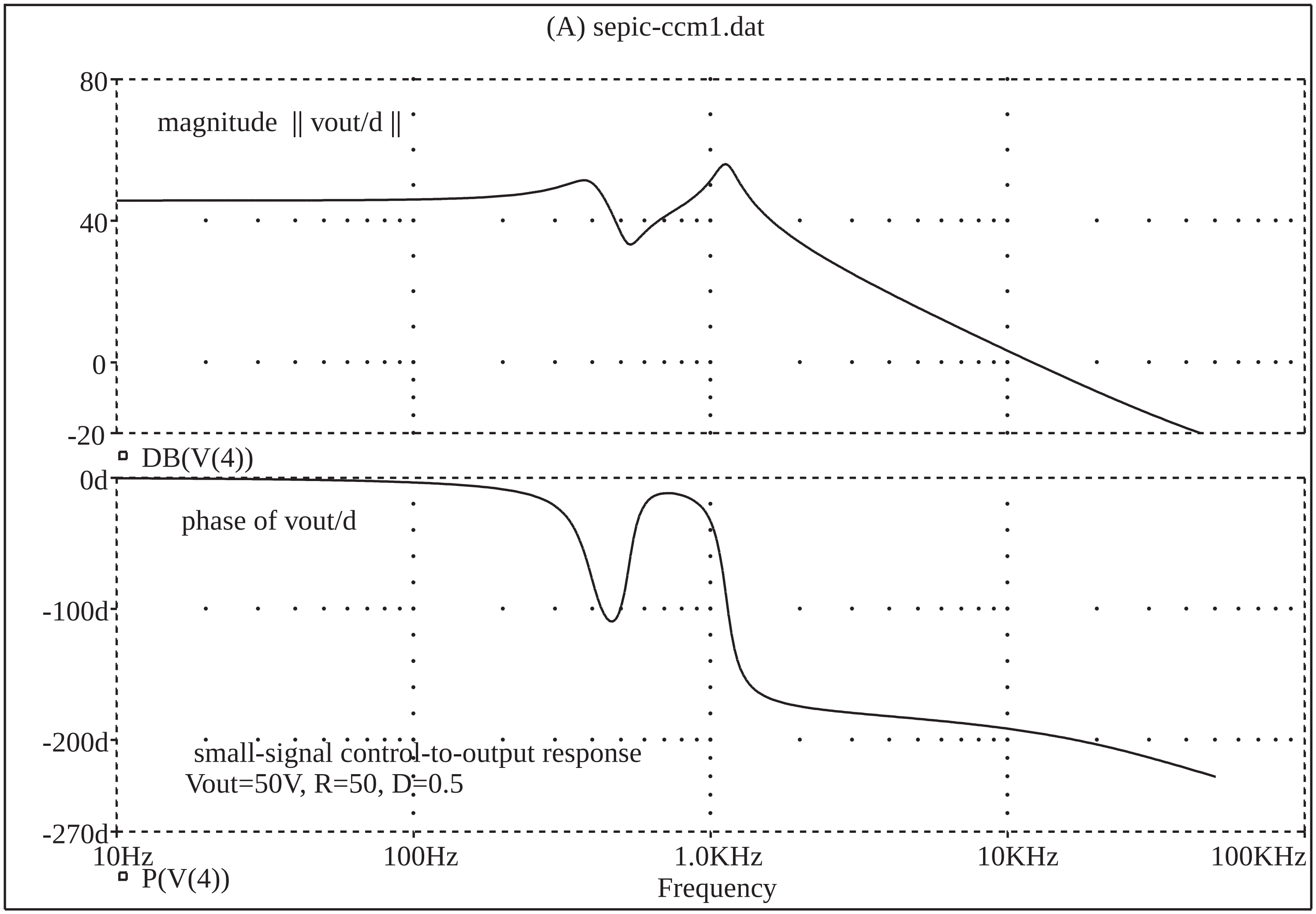

```
**********************************************************
* MODEL: ccm1
* Application: two-switch PWM converters
* Limitations: ideal switches, CCM only, no transformer
**********************************************************
* Parameters: none
**********************************************************
* Nodes:
* 1: transistor+ (D)
* 2: transistor- (S)
* 3: diode cathode (K)
* 4: diode anode (A)
* 5: duty ratio (duty)
**********************************************************
.subckt ccm1 1  2  3  4  5
Et 1 2 value={(1-v(5))*v(3,4)/v(5)}
Gd 4 3 value={(1-v(5))*i(Et)/v(5)}
.ends
**********************************************************
```

Sepic converter example using ccm1 model



Objective: generate small-signal control-to-output frequency responses

Sepic converter example using ccm1 model: small-signal control-to-output response

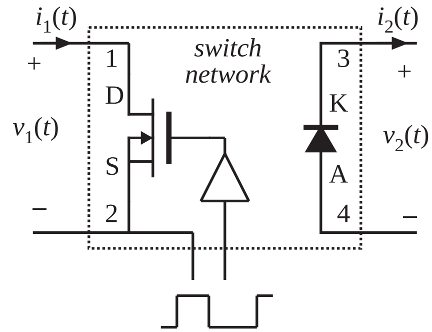


Comments

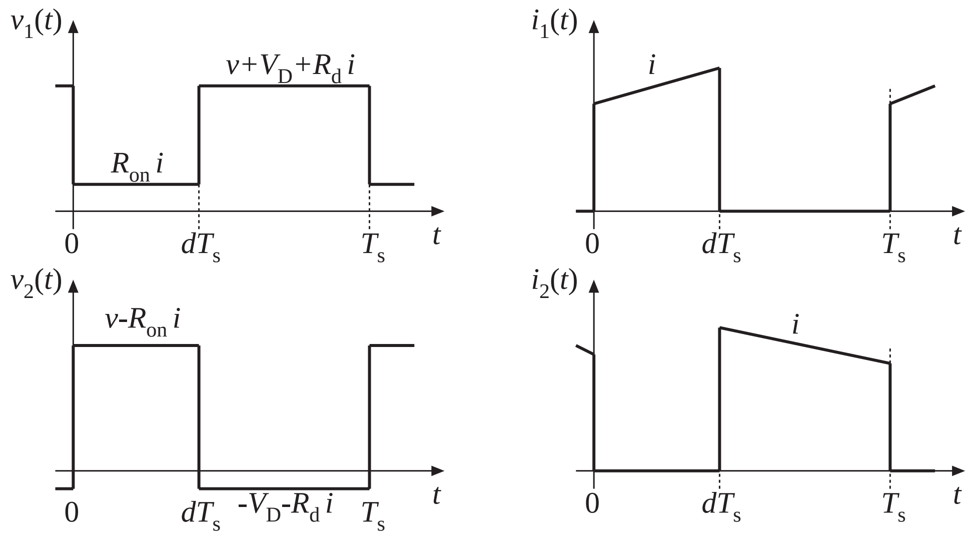
- Subcircuit **ccm1** is implementation of a large-signal, nonlinear averaged model of the switch network
- Averaged circuit model of the converter is obtained simply by replacing switching devices with the averaged-switch subcircuit model
- Linearization and AC small-signal analysis are performed by the simulator
- Small-signal dynamic responses can be easily generated for different operating points or different sets of parameter values

CCM Averaged Switch Model That Includes Conduction Losses

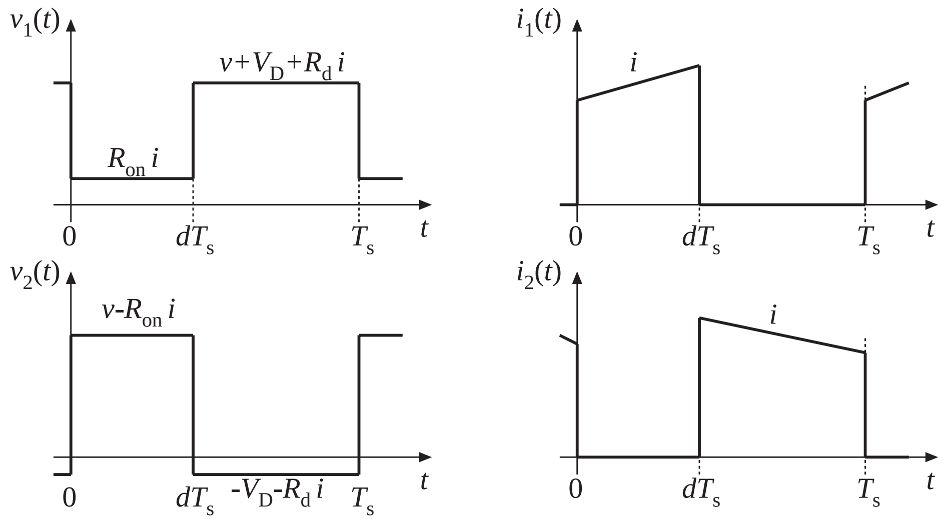
- MOS transistor model: on-resistance R_{ON}
- Diode model: constant forward voltage drop V_D in series with R_d resistance
- Switch network



- Waveforms



CCM Averaged Switch Model ccm2 That Includes Conduction Losses



$$\langle i_1 \rangle_{T_s} = d \langle i \rangle_{T_s}$$

$$\langle i_2 \rangle_{T_s} = (1-d) \langle i \rangle_{T_s}$$

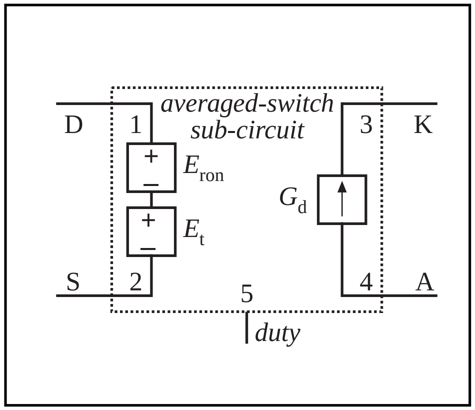
$$\boxed{\langle i_2 \rangle_{T_s} = \frac{1-d}{d} \langle i_1 \rangle_{T_s}}$$

$$\langle v_1 \rangle_{T_s} = dR_{on} \langle i \rangle_{T_s} + (1-d)\left(\langle v \rangle_{T_s} + V_D + R_d i\right)$$

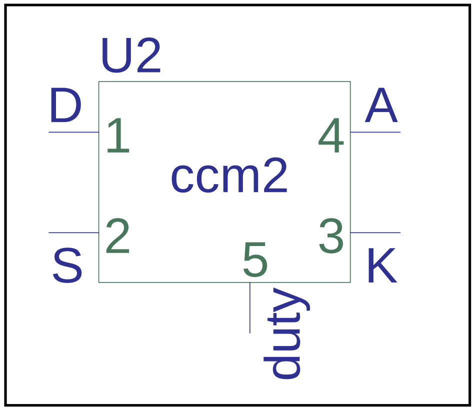
$$\langle v_1 \rangle_{T_s} + \langle v_2 \rangle_{T_s} = \langle v \rangle_{T_s}$$

$$\boxed{\langle v_1 \rangle_{T_s} = \frac{R_{on} \langle i_1 \rangle_{T_s}}{d} + \frac{(1-d) R_d \langle i_1 \rangle_{T_s}}{d^2} + \frac{1-d}{d}\left(\langle v_1 \rangle_{T_s} + V_D\right)}$$

CCM Averaged-Switch Model PSpice Implementation: ccm2

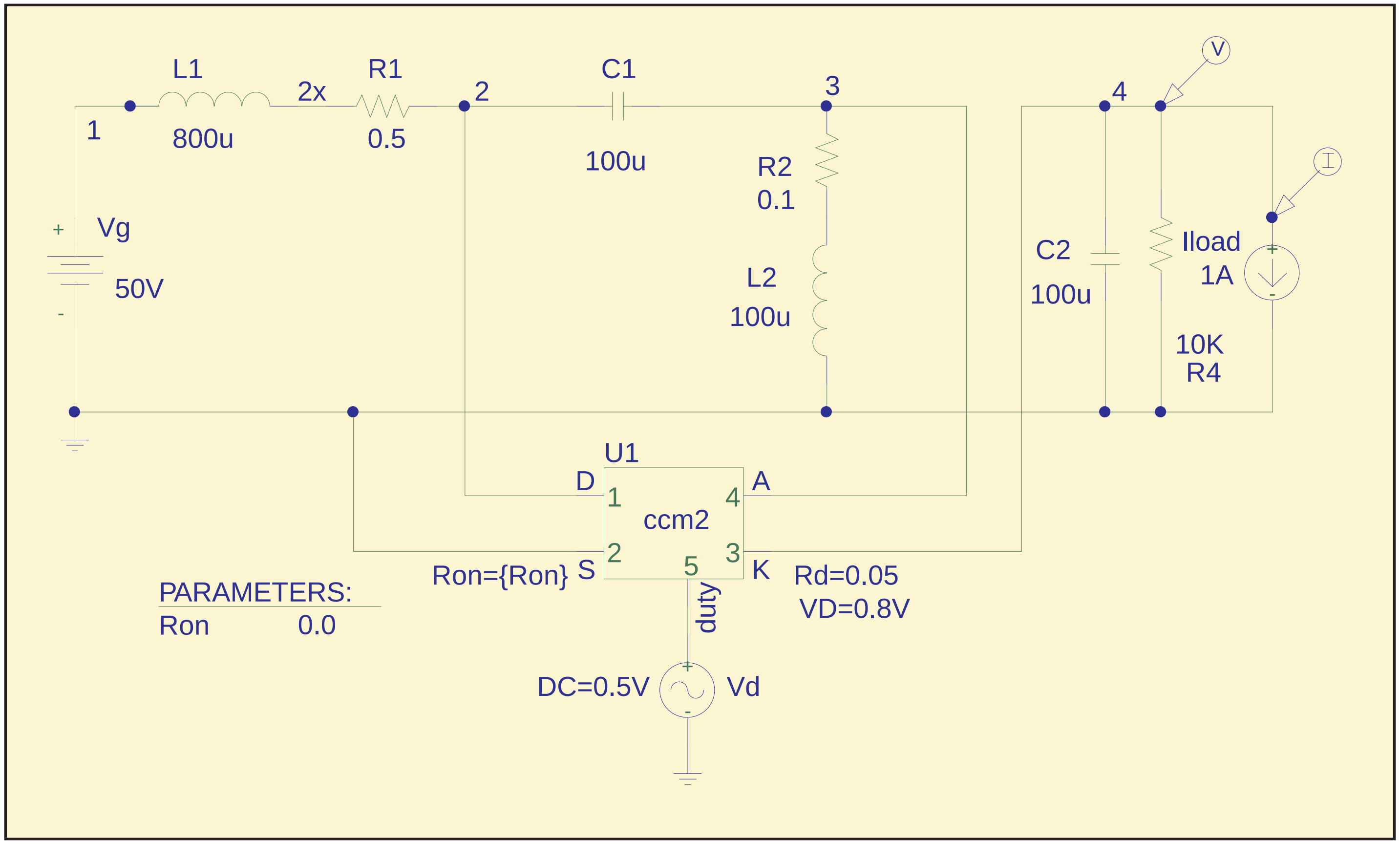


Subcircuit implementation



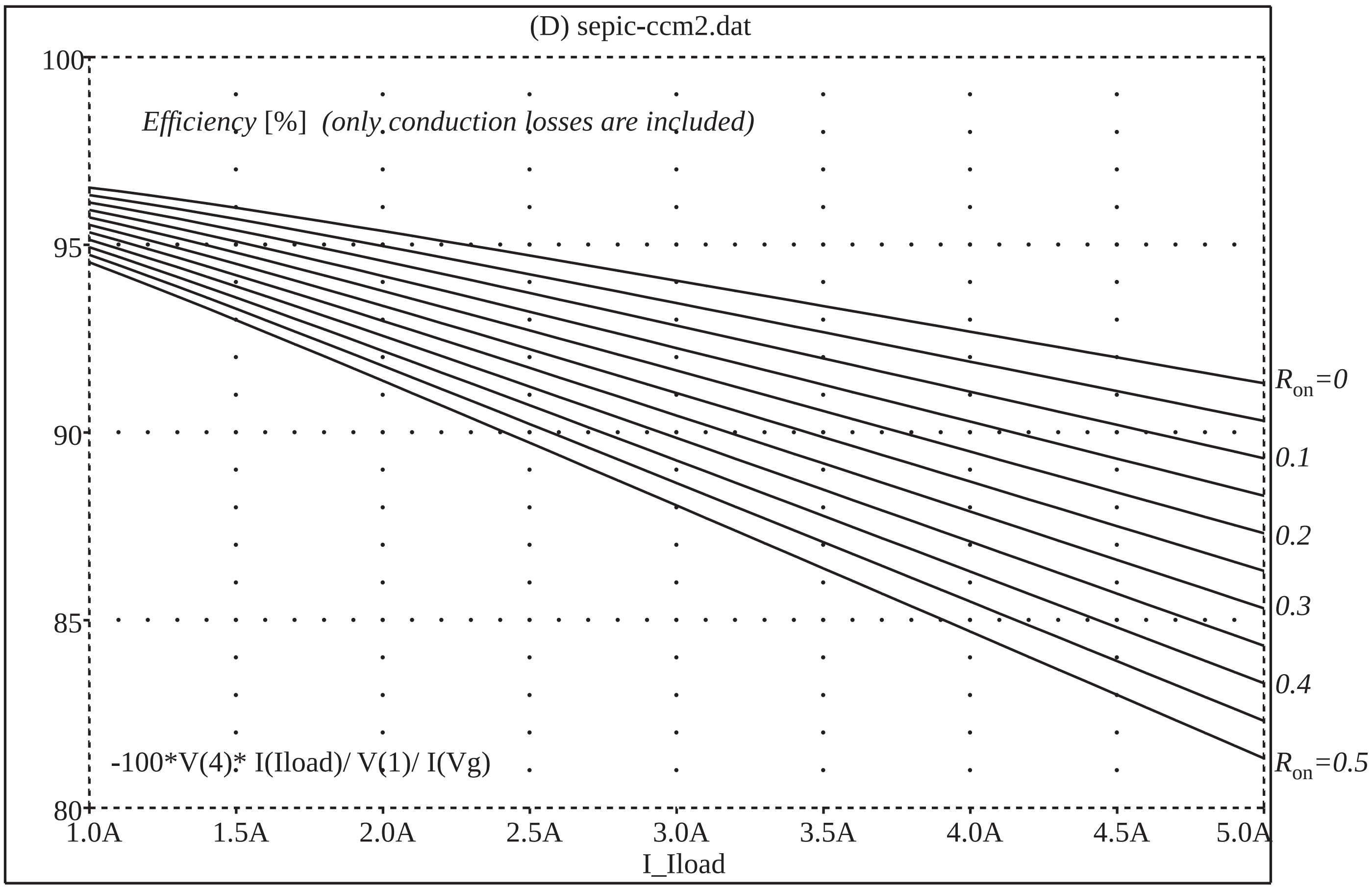

```
*******************************************************
* MODEL: ccm2
* Application: two-switch PWM converters, includes
*            conduction losses due to Ron, VD, Rd
* Limitations: CCM only, no transformer
*******************************************************
* Parameters:
*     Ron=transistor on resistance
*     VD=diode forward voltage drop (constant)
*     Rd=diode on resistance
*******************************************************
* Nodes: (same as in ccm1)
*******************************************************
.subckt ccm2 1  2  3  4  5
+params: Ron=0 VD=0 Rd=0
Eron 1 1x value={i(Et)*(Ron+(1-v(5))*Rd/v(5))/v(5)}
Et 1x 2 value={(1-v(5))*(v(3,4)+VD)/v(5)}
Gd 4 3 value={(1-v(5))*i(Et)/v(5)}
.ends
*******************************************************
```

Sepic converter example using ccm2 model



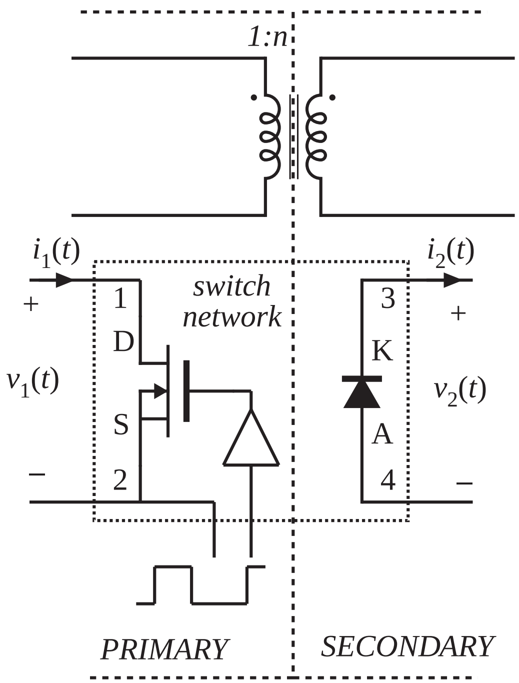
Objective: find converter efficiency as a function of the transistor on-resistance, for a range of loads

Sepic converter example using ccm2 model: Efficiency vs. load current and Ron

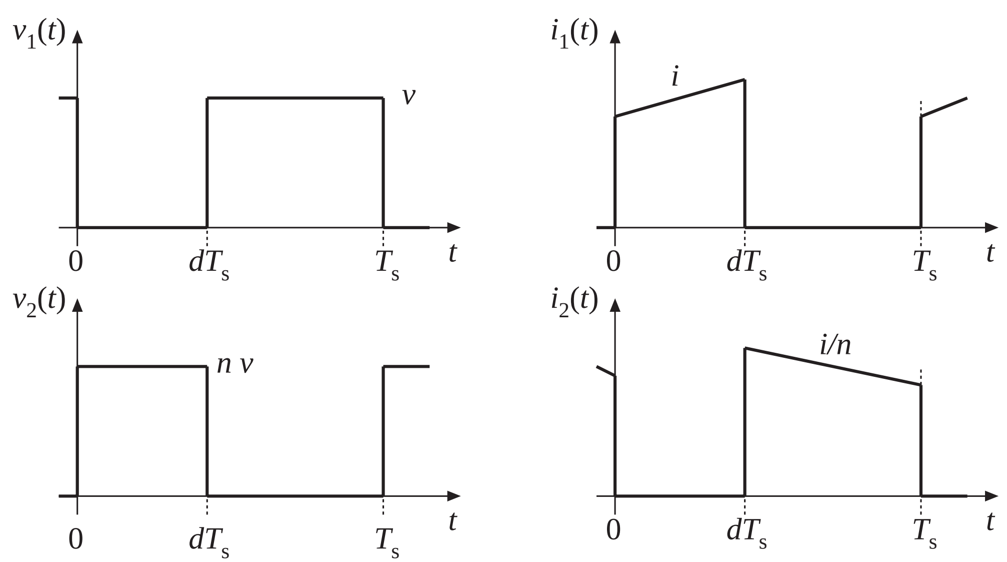


CCM Averaged Switch Model ccm3
For Converters With Isolation Transformer

Switch network



Waveforms

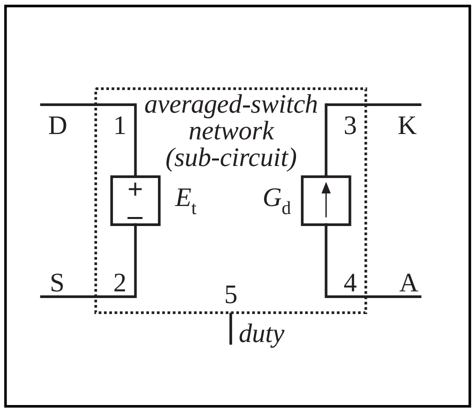


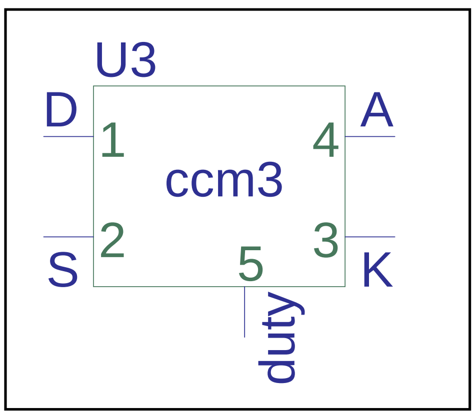
$$\left\langle v_1 \right\rangle_{T_s} = \frac{1-d}{nd} \left\langle v_2 \right\rangle_{T_s}$$

$$\left\langle i_2 \right\rangle_{T_s} = \frac{1-d}{nd} \left\langle i_1 \right\rangle_{T_s}$$

- Converters: Flyback, Cuk, Sepic, Inverse Sepic (Zeta), with isolation transformer

CCM Averaged-Switch Model
PSpice Implementation: ccm3



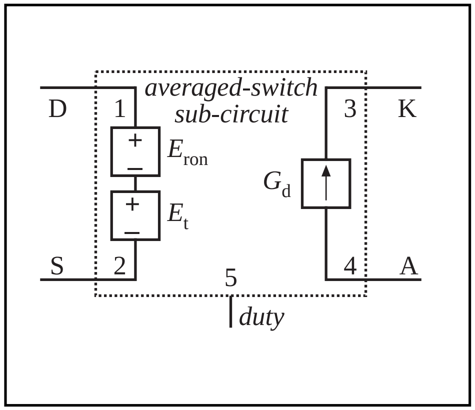



```
**********************************************************
* MODEL: ccm3
* Application: two-switch PWM converters,
*              with (possibly) transformer
* Limitations: ideal switches, CCM only
**********************************************************
* Parameters:
*    n=transformer turns ratio 1:n (primary:secondary)
**********************************************************
* Nodes: (same as in ccm1)
**********************************************************
.subckt ccm3 1  2  3  4  5
+params: n=1
Et 1 2 value={(1-v(5))*v(3,4)/v(5)/n}
Gd 4 3 value={(1-v(5))*i(Et)/v(5)/n}
.ends
**********************************************************
```

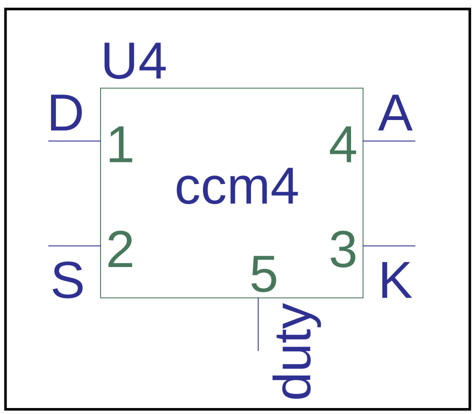
CCM Averaged Switch Model ccm4 (Conduction Losses and Isolation Transformer)

- Combined **ccm2** and **ccm3** averaged-switch models
- Parameters:
  - Transistor on resistance R_{on}
  - Diode forward voltage drop V_D
  - Diode on resistance R_d
  - Transformer turns ratio n
- A general model implementation valid for all two-switch converters operating in CCM

CCM Averaged-Switch Model
PSpice Implementation: ccm4



Subcircuit implementation



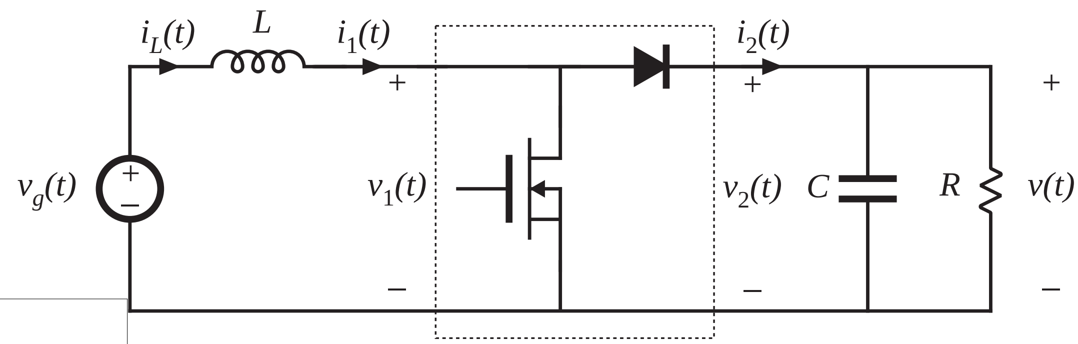

```
* MODEL: ccm4
* Application: two-switch PWM converters, includes
*             conduction losses due to Ron, VD, Rd
*             and (possibly) transformer
* Limitations: CCM only
****************************************************************
* Parameters:
*      Ron=transistor on resistance
*      VD=diode forward voltage drop (constant)
*      Rd=diode on resistance
*      n=transformer turns ratio 1:n (primary:secondary)
****************************************************************
* Nodes: (same as in ccm1)
****************************************************************
.subckt ccm4 1  2  3  4  5
+params: Ron=0 VD=0 Rd=0 n=1
Eron 1 1x value={i(Et)*(Ron+(1-v(5))*Rd/n/n/v(5))/v(5)}
Et 1x 2 value={(1-v(5))*(v(3,4)+VD)/v(5)/n}
Gd 4 3 value={(1-v(5))*i(Et)/v(5)/n}
.ends
```

Averaged-Switch Modeling Exercise: Include Switching Loss

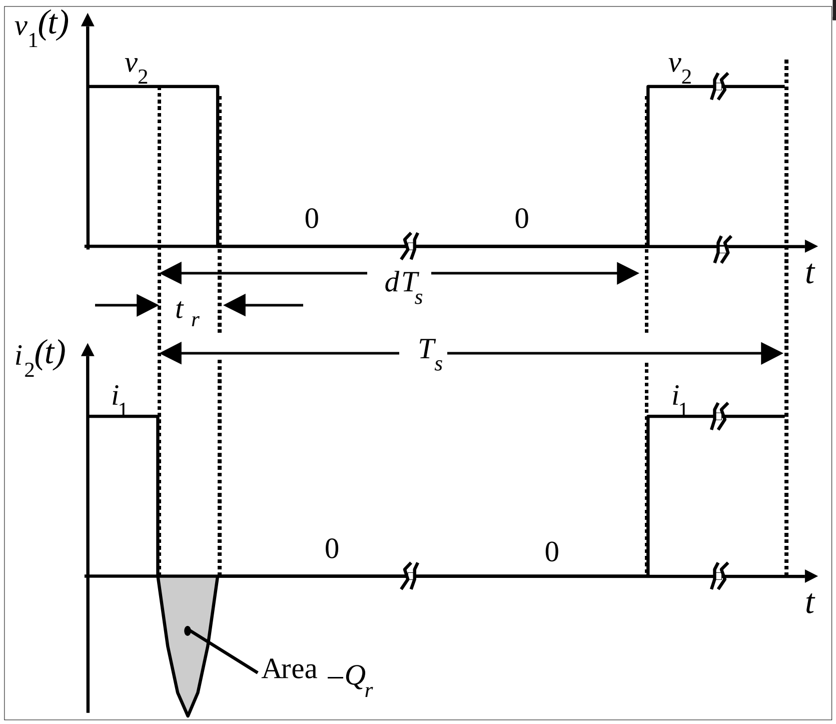
- Use averaged-switch modeling approach to construct an averaged model that includes switching losses
- Loss mechanism example: diode reverse recovery

Modeling switching loss

Example: diode stored charge in boost converter



Waveforms:



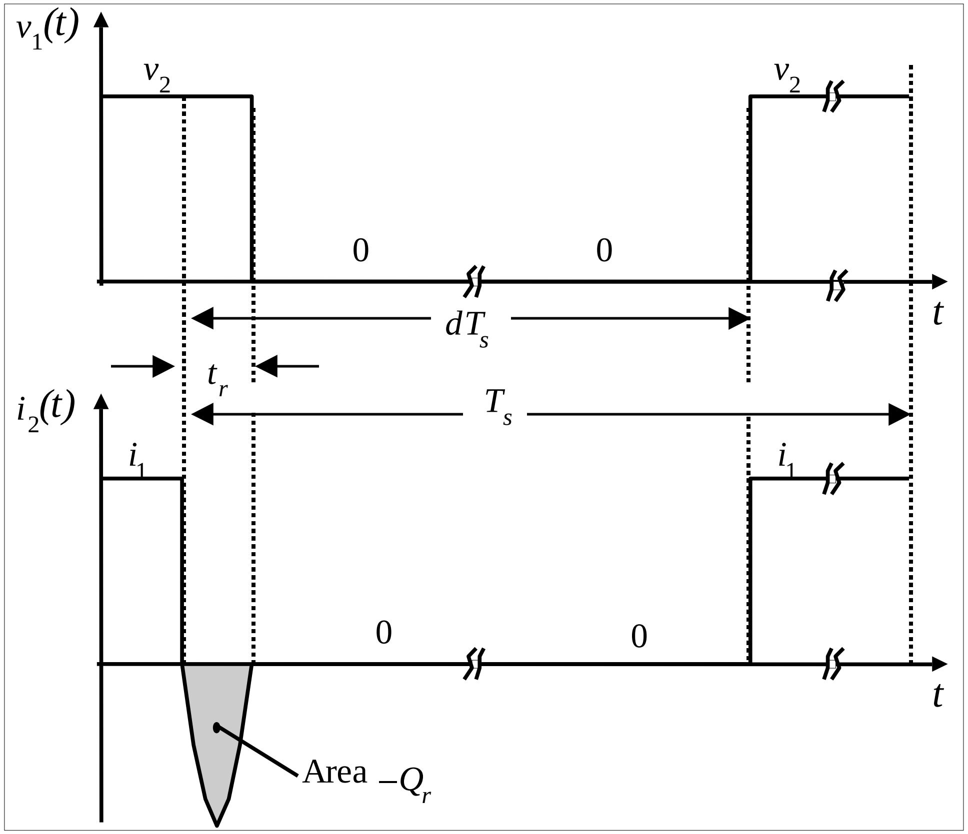
- Other switching loss mechanisms are ignored in this example; one can include other losses if desired, using a similar procedure
- Determine averaged terminal waveforms of switch network
- Construct averaged equivalent circuit model

Expressions for average terminal waveforms

Boost converter, switching loss example

$$\left\langle v_1(t) \right\rangle_{T_s} = \frac{1}{T_s}\left(\left(1-d\right)T_s + t_r\right)\left\langle v_2(t) \right\rangle_{T_s}$$

$$\left\langle i_2(t) \right\rangle_{T_s} = \left(1-d\right)\left\langle i_1(t) \right\rangle_{T_s} - \frac{Q_r}{T_s}$$



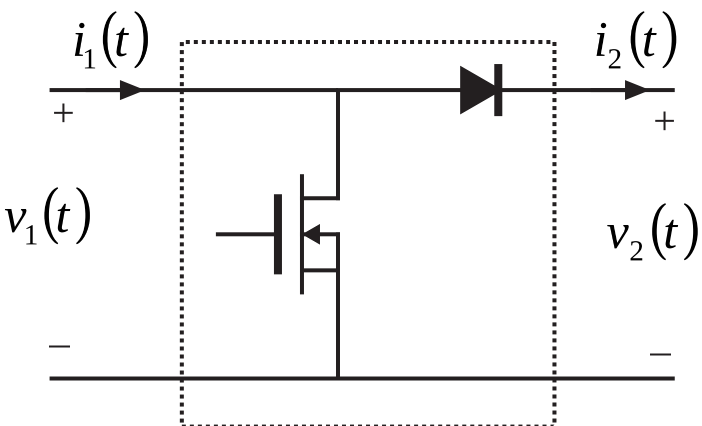
t_r = diode reverse recovery time

Q_r = diode recovered charge

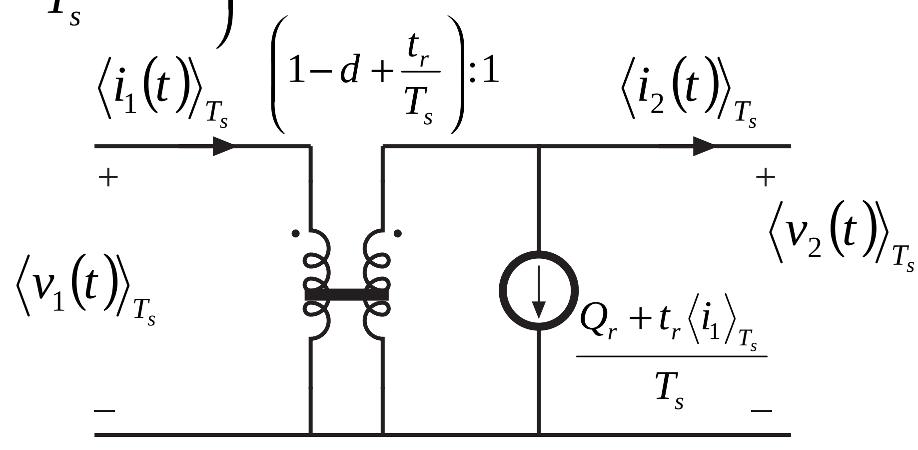
Averaged equivalent circuit
of switch network

$$\left\langle v_1(t) \right\rangle_{T_s} = \left(1 - d + \frac{t_r}{T_s}\right) \left\langle v_2(t) \right\rangle_{T_s}$$

$$\left\langle i_2(t) \right\rangle_{T_s} = \left(1 - d + \frac{t_r}{T_s}\right) \left\langle i_1(t) \right\rangle_{T_s} - \left(\frac{Q_r + t_r \left\langle i_1(t) \right\rangle_{T_s}}{T_s} \right)$$



switch network

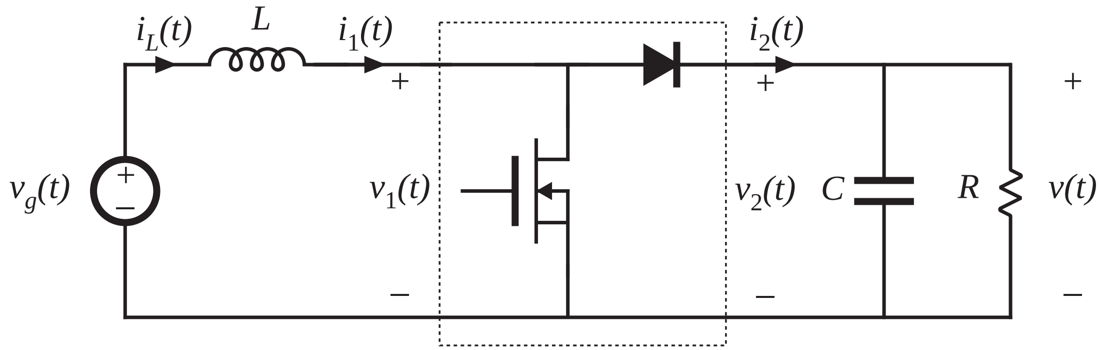


averaged switch model

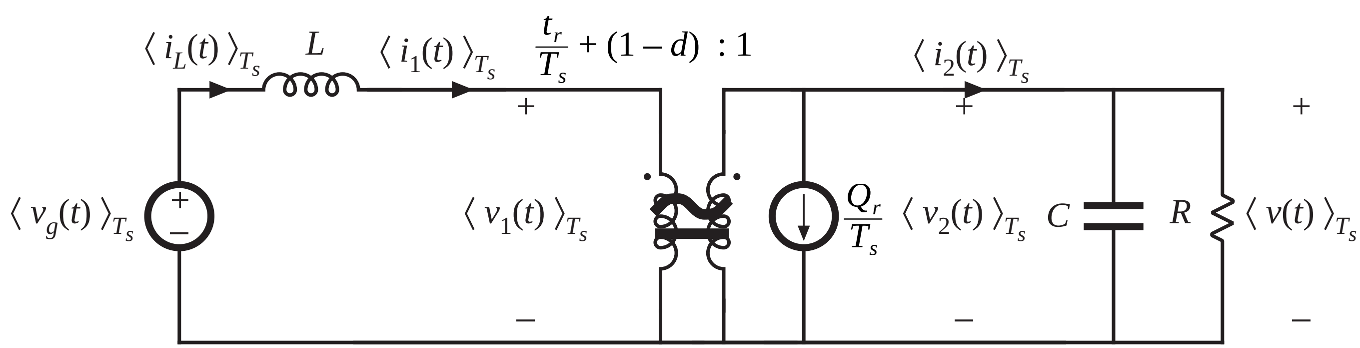
- Diode reverse recovery time affects conversion ratio
- Stored charge leads to power loss, modeled by current sink

Insert averaged switch model into converter circuit

Original converter



Averaged model



Efficiency Analysis

Boost converter, switching loss example

$$\eta = \frac{P_{out}}{P_{in}} = \frac{VI_2}{V_g I_1}$$

$$I_1 = \frac{I_2 + \frac{Q_r}{T_s}}{1 - D} \qquad V = \frac{V_g}{\frac{t_r}{T_s} + 1 - D}$$

$$\boxed{\eta = \frac{VI_2}{V_g I_1} = \left(\frac{1-D}{1-D+\frac{t_r}{T_s}} \right) \left(\frac{I_2}{I_2 + \frac{Q_r}{T_s}} \right) = \left(\frac{1}{1+\frac{t_r}{(1-D)T_s}} \right) \left(\frac{1}{1+\frac{Q_r}{I_{load} T_s}} \right)}$$

Efficiency due to diode reverse recovery. Other switching loss mechanisms can be included using a similar procedure.

Summary of Part 2

- Basic idea of average-switch modeling:

  Define a switch network, containing all of the converter switching elements

  Average terminal waveforms over a switching period

  Use controlled sources with values equal to average of the switch network terminal waveforms

  The result is a large-signal, nonlinear, time-invariant model that can be inserted back into the converter network
- The choices of the switch network and the independent terminal waveforms are not unique - there are many ways to construct averaged switch models
- Averaged-switch model (suitable for circuit analysis or simulation) yields predictions of converter steady-state and low-frequency dynamic properties
- Next: apply the averaged-switch modeling approach to other cases of interest.

3. Averaged switch modeling of PWM converters operating in the discontinuous conduction mode

- **Averaged switch model in DCM**
- **Using averaged-switch model to predict converter steady-state characteristics and small-signal dynamics in DCM**
- **Combined CCM/DCM averaged switch model**
- **PSpice implementation of combined CCM/DCM models**
  - ideal switches (ccm-dcm1)
  - ideal switches in converters with isolation transformer (ccm-dcm2)
- **Application examples:**
  - comparison of transient simulation results in a SEPIC example using (1) switching circuit model and (2) averaged model
  - small-signal dynamic responses of a flyback converter operating in CCM or DCM
  - more converter examples using averaged-switch subcircuits

Change in characteristics at the CCM/DCM boundary

- Steady-state output voltage becomes strongly load-dependent
- Simpler dynamics: one pole and the RHP zero are moved to very high frequency, and can normally be ignored
- Traditionally, boost and buck-boost converters are designed to operate in DCM at full load
- All converters may operate in DCM at light load

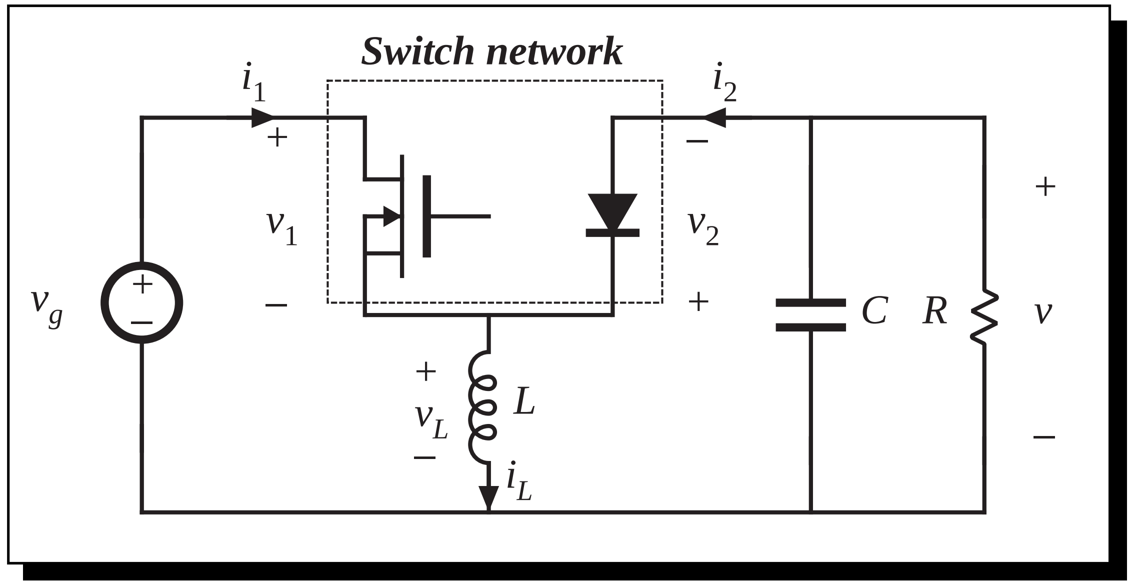
So we need equivalent circuits that model the steady-state and small-signal ac models of converters operating in DCM

The averaged switch approach yields an intuitive result that is relatively easy to solve

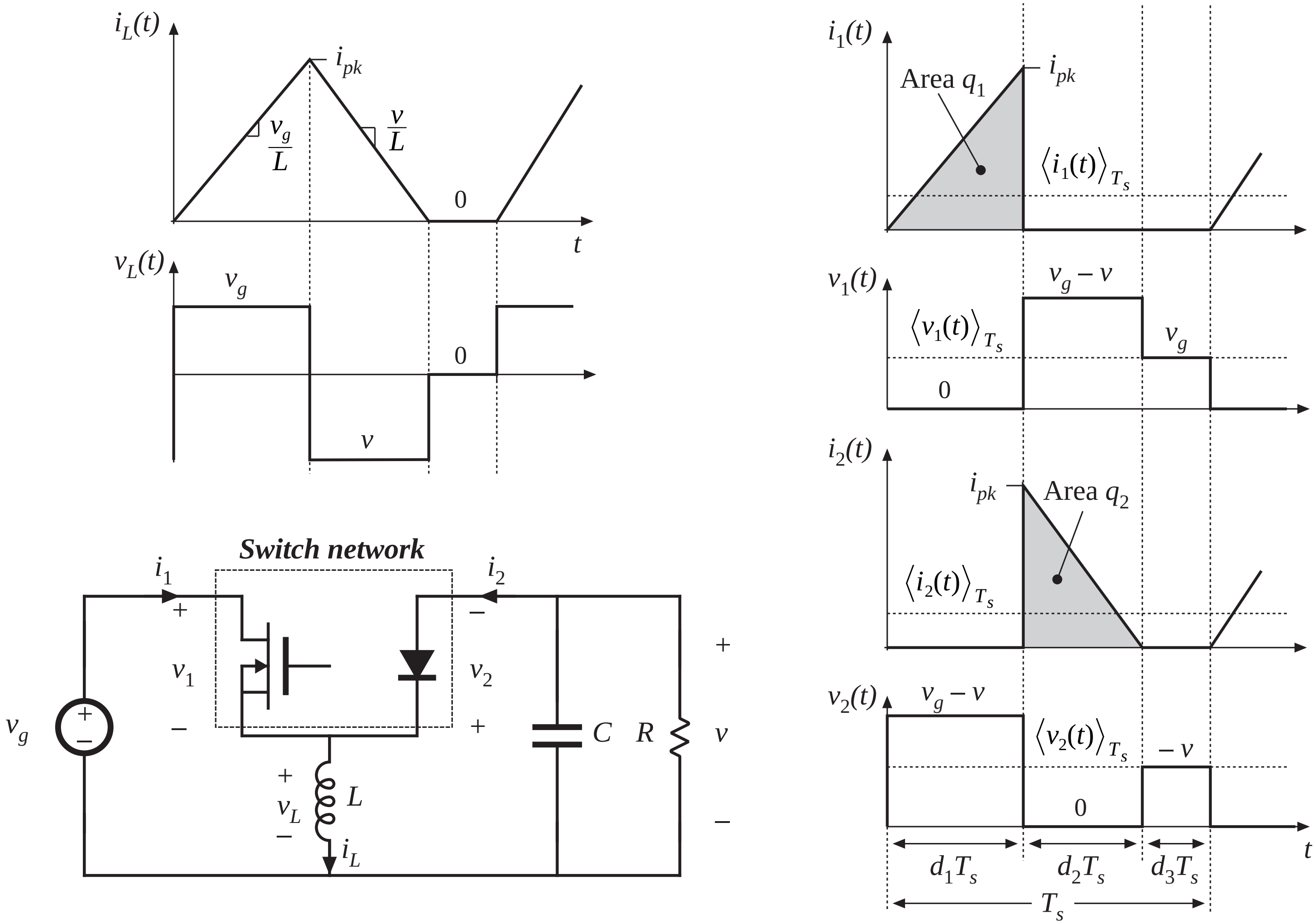
Derivation of DCM averaged switch model

Buck-boost example

- Define switch terminal quantities v_1, i_1, v_2, i_2, as shown
- Let us find the averaged quantities $\langle v_1 \rangle$, $\langle i_1 \rangle$, $\langle v_2 \rangle$, $\langle i_2 \rangle$, for operation in DCM, and determine the relations between them



DCM waveforms



Basic DCM equations

Peak inductor current:

$$i_{pk} = \frac{v_g}{L} d_1 T_s$$

Average inductor voltage:

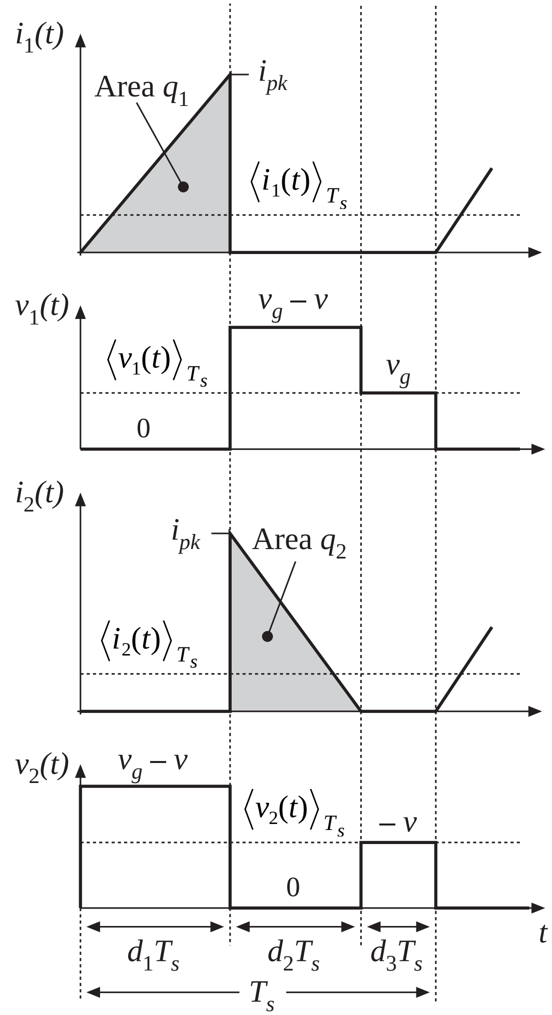
$$\left\langle v_L(t) \right\rangle_{T_s} = d_1 \left\langle v_g(t) \right\rangle_{T_s} + d_2 \left\langle v(t) \right\rangle_{T_s} + d_3 \cdot 0$$

In DCM, the diode switches off when the inductor current reaches zero. Hence, $i(0) = i(T_s) = 0$, and the average inductor voltage is zero. This is true even during transients.

$$\left\langle v_L(t) \right\rangle_{T_s} = d_1(t) \left\langle v_g(t) \right\rangle_{T_s} + d_2(t) \left\langle v(t) \right\rangle_{T_s} = 0$$

Solve for d_2:

$$d_2(t) = - d_1(t) \frac{\left\langle v_g(t) \right\rangle_{T_s}}{\left\langle v(t) \right\rangle_{T_s}}$$



Average switch network terminal voltages

Average the $v_1(t)$ waveform:

$$\left\langle v_1(t) \right\rangle_{T_s} = d_1(t) \cdot 0 + d_2(t) \left(\left\langle v_g(t) \right\rangle_{T_s} - \left\langle v(t) \right\rangle_{T_s} \right) + d_3(t) \left\langle v_g(t) \right\rangle_{T_s}$$

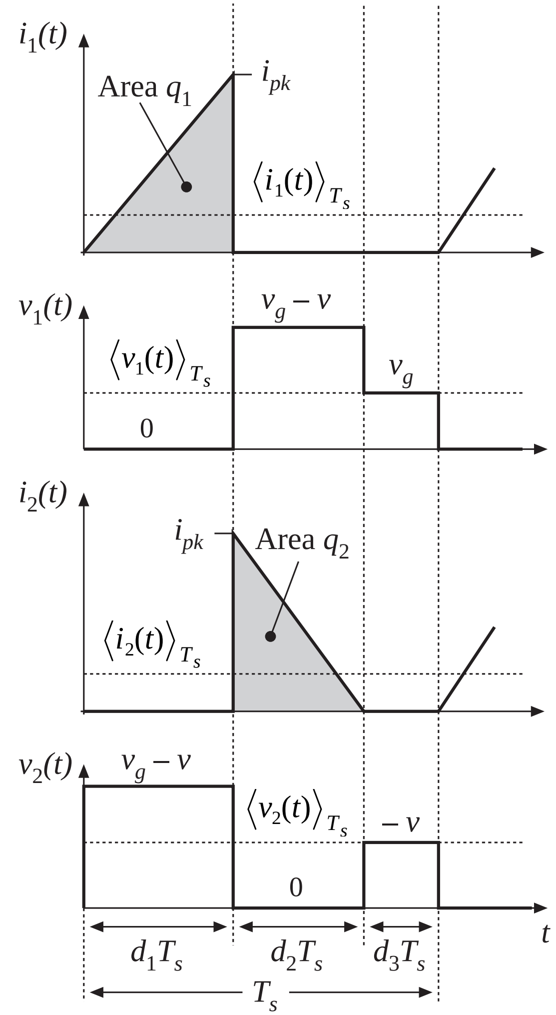
Eliminate d_2 and d_3:

$$\left\langle v_1(t) \right\rangle_{T_s} = \left\langle v_g(t) \right\rangle_{T_s}$$

Similar analysis for $v_2(t)$ waveform leads to

$$\left\langle v_2(t) \right\rangle_{T_s} = d_1(t) \left(\left\langle v_g(t) \right\rangle_{T_s} - \left\langle v(t) \right\rangle_{T_s} \right) + d_2(t) \cdot 0 + d_3(t) \left(- \left\langle v(t) \right\rangle_{T_s} \right)$$

$$= - \left\langle v(t) \right\rangle_{T_s}$$



Average switch network terminal currents

Average the $i_1(t)$ waveform:

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} i_1(t) dt = \frac{q_1}{T_s}$$

The integral q_1 is the area under the $i_1(t)$ waveform during first subinterval. Use triangle area formula:

$$q_1 = \int_t^{t+T_s} i_1(t) dt = \frac{1}{2} \left(d_1 T_s \right) \left(i_{pk} \right)$$

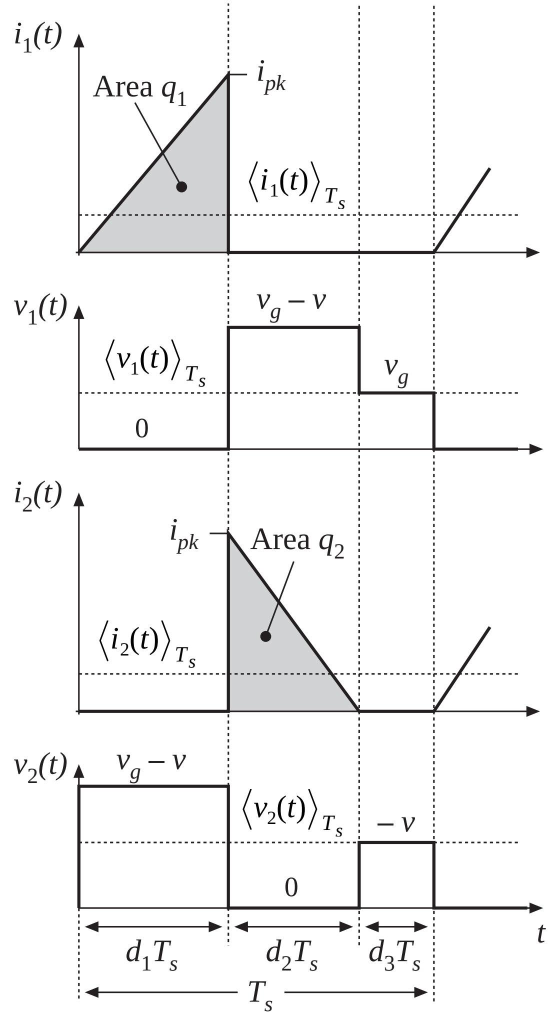
Eliminate i_{pk}:

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{d_1^2(t)\, T_s}{2L} \left\langle v_1(t) \right\rangle_{T_s}$$

Note $\langle i_1(t) \rangle_{Ts}$ is not equal to $d \, \langle i_L(t) \rangle_{Ts}$!

Similar analysis for $i_2(t)$ waveform leads to

$$\left\langle i_2(t) \right\rangle_{T_s} = \frac{d_1^2(t)\, T_s}{2L} \frac{\left\langle v_1(t) \right\rangle_{T_s}^2}{\left\langle v_2(t) \right\rangle_{T_s}}$$

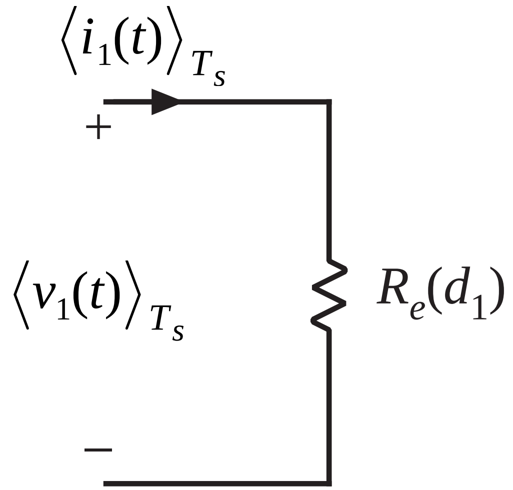


Input port: Averaged equivalent circuit

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{d_1^2(t)\, T_s}{2L} \left\langle v_1(t) \right\rangle_{T_s}$$

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{\left\langle v_1(t) \right\rangle_{T_s}}{R_e(d_1)}$$

$$R_e(d_1) = \frac{2L}{d_1^2\, T_s}$$

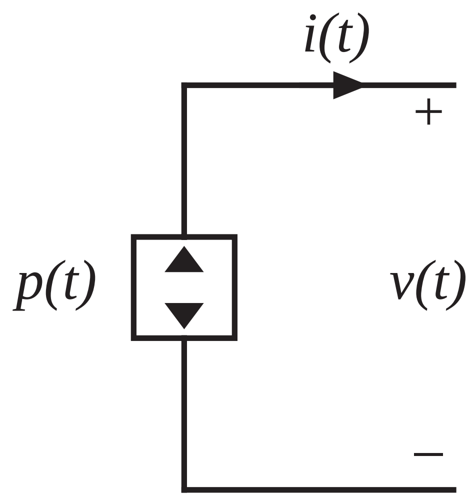


Low-frequency components of input port waveforms obey Ohm’s law

Output port: Averaged equivalent circuit

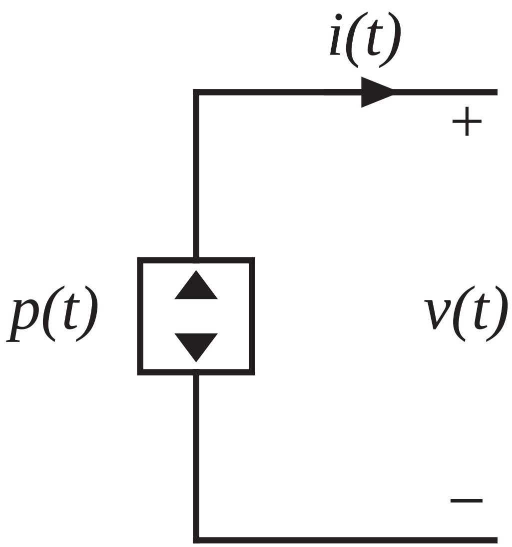
$$\left\langle i_2(t)\right\rangle_{T_s} = \frac{d_1^2(t)\,T_s}{2L}\,\frac{\left\langle v_1(t)\right\rangle_{T_s}^2}{\left\langle v_2(t)\right\rangle_{T_s}}$$

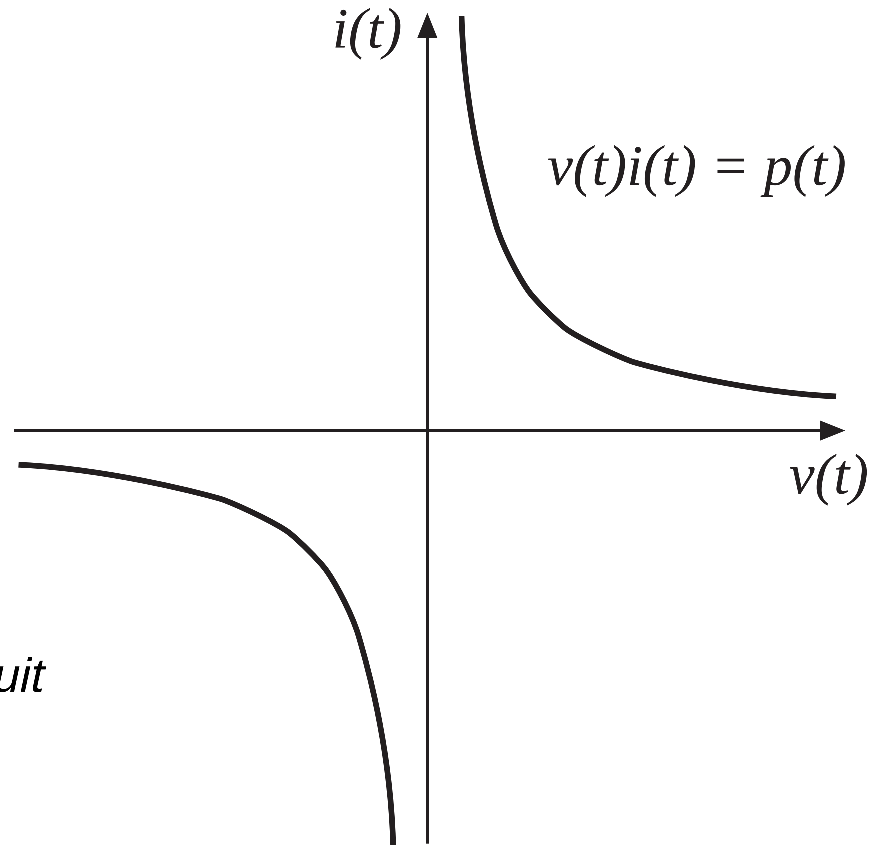
$$\left\langle i_2(t)\right\rangle_{T_s}\left\langle v_2(t)\right\rangle_{T_s} = \frac{\left\langle v_1(t)\right\rangle_{T_s}^2}{R_e(d_1)} = \left\langle p(t)\right\rangle_{T_s}$$



- Output port is a source of power $p(t)$
- Power $p(t)$ is independent of load characteristics
- Power $p(t)$ is dependent on (equal to) the power apparently consumed by the switch network input port

The dependent power source





- *Must avoid open- and short-circuit connections of power sources*
- *Power sink: negative* $p(t)$

How the power source arises in lossless two-port networks

In a lossless two-port network without internal energy storage: instantaneous input power is equal to instantaneous output power

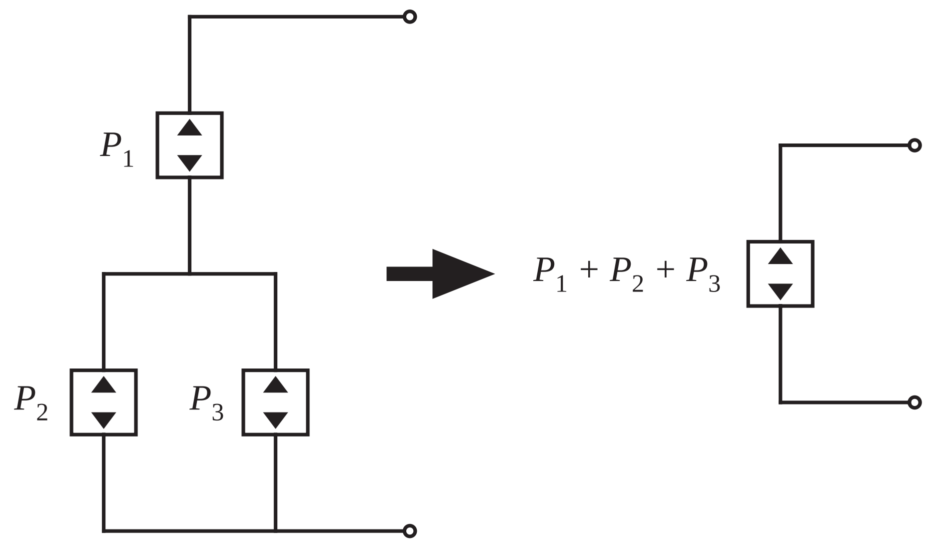
In all but a small number of special cases, the instantaneous power throughput is dependent on the applied external source and load

If the instantaneous power depends only on the external elements connected to one port, then the power is not dependent on the characteristics of the elements connected to the other port. The other port becomes a source of power, equal to the power flowing through the first port

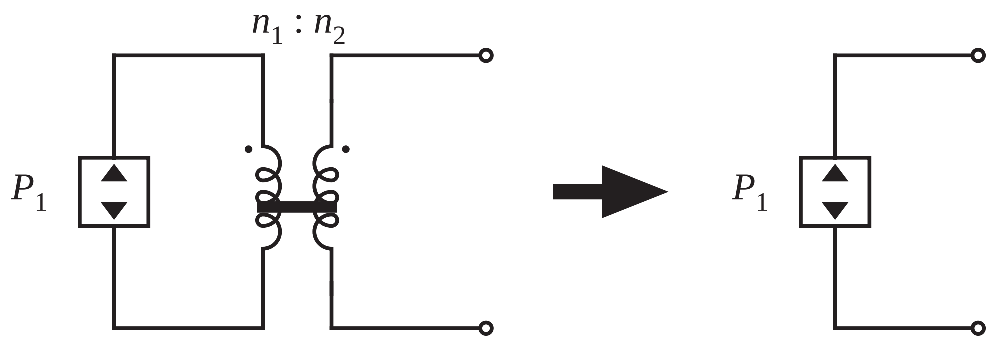
A power source (or power sink) element is obtained

Properties of power sources

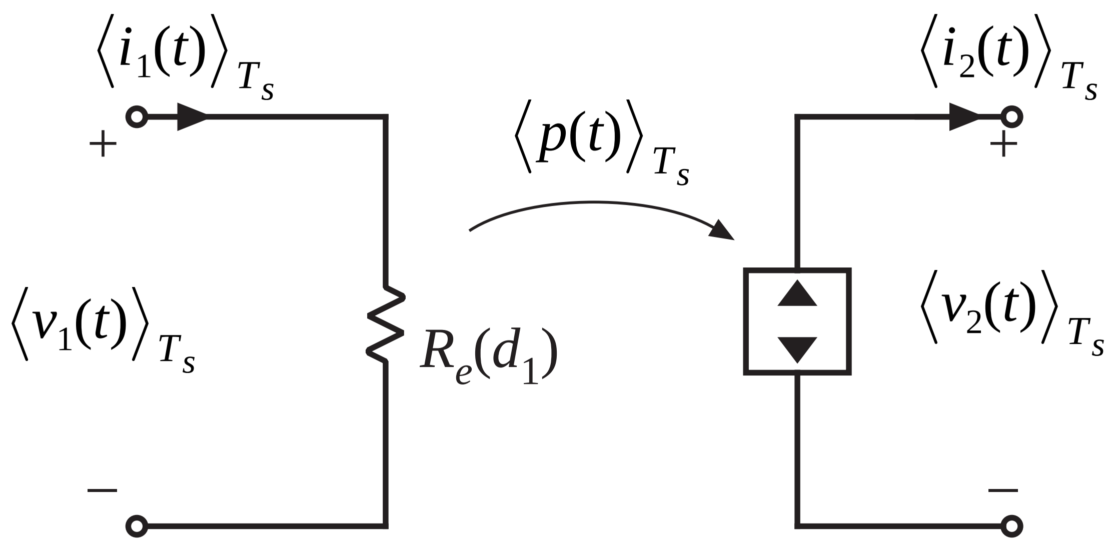
Series and parallel connection of power sources



Reflection of power source through a transformer



The loss-free resistor (LFR)



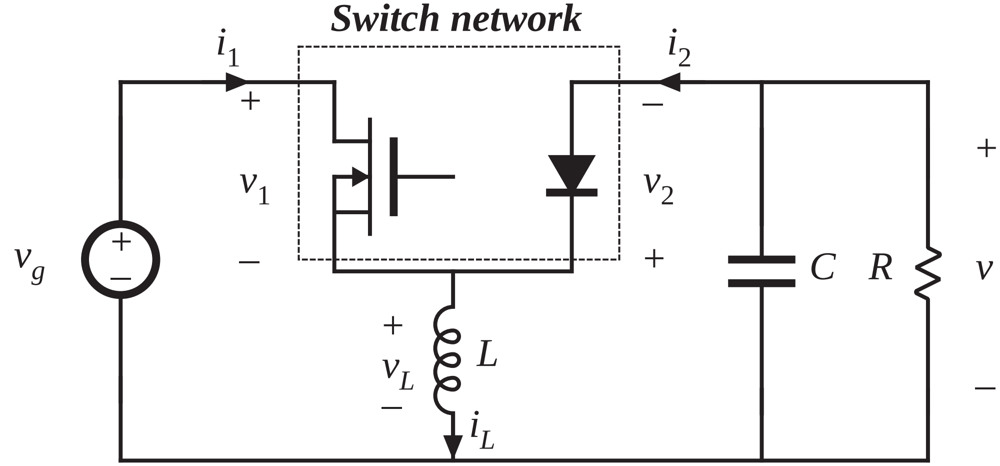
A two-port lossless network

Input port obeys Ohm's Law

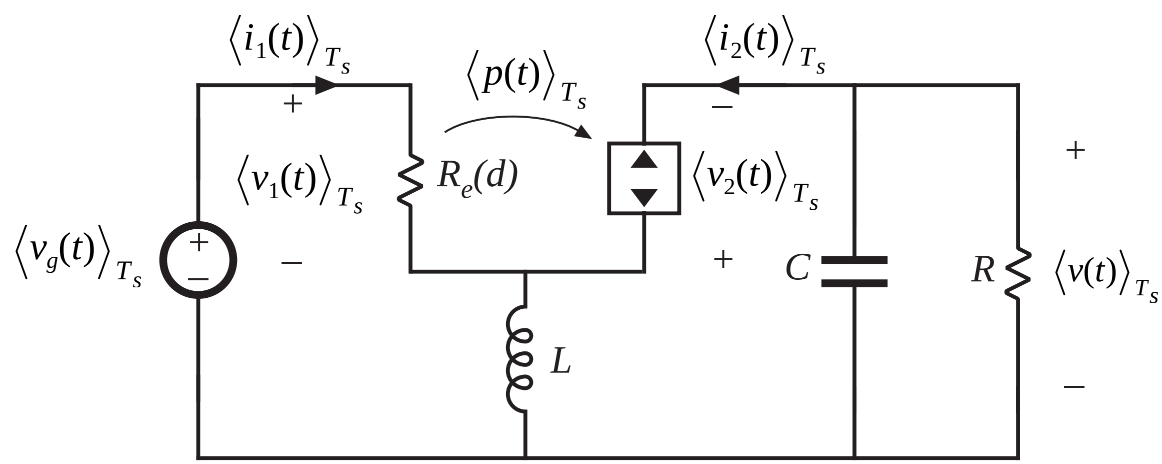
Power entering input port is transferred to output port

Averaged switch model: buck-boost example

Original circuit



Averaged model

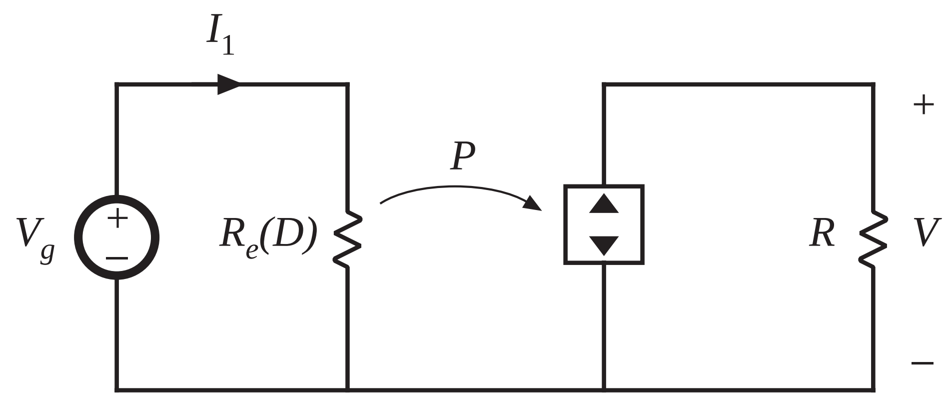


Solution of averaged model: steady state

Let

$L \rightarrow$ short circuit

$C \rightarrow$ open circuit



Converter input power:

$$P = \frac{V_g^2}{R_e}$$

Converter output power:

$$P = \frac{V^2}{R}$$

Equate and solve:

$$P = \frac{V_g^2}{R_e} = \frac{V^2}{R}$$

$$\frac{V}{V_g} = \pm \sqrt{\frac{R}{R_e}}$$

Steady-state LFR solution

$$\frac{V}{V_g} = \pm \sqrt{\frac{R}{R_e}}$$

is a general result, for any system that can be modeled as an LFR.

For the buck-boost converter, we have

$$R_e(D) = \frac{2L}{D^2 T_s}$$

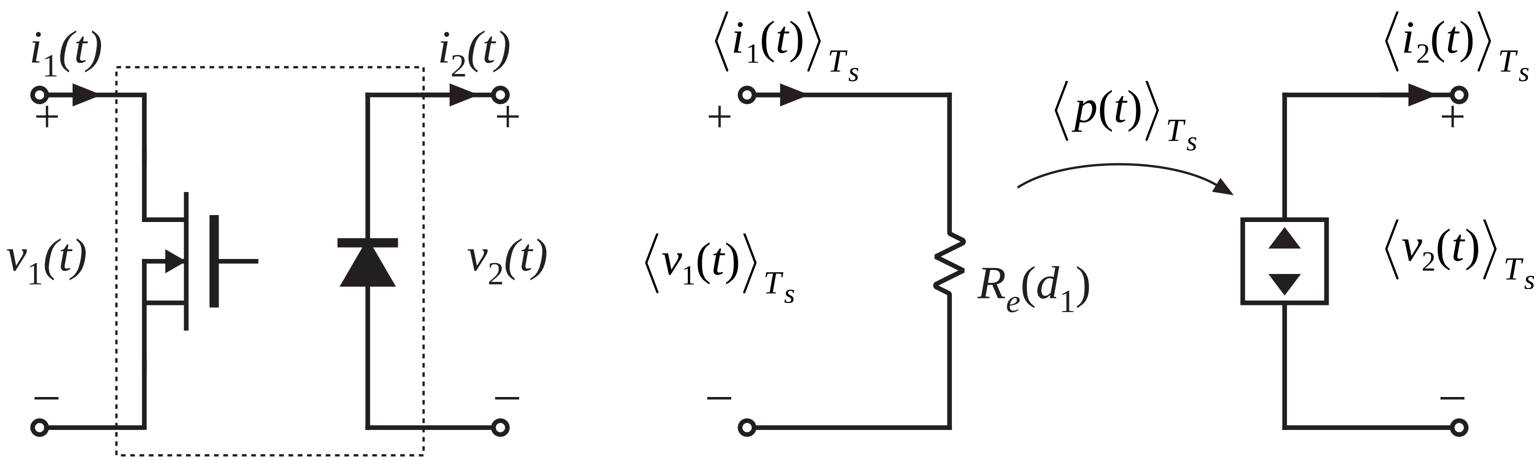
Eliminate R_e:

$$\frac{V}{V_g} = -\sqrt{\frac{D^2 T_s R}{2L}} = -\frac{D}{\sqrt{K}}$$

which agrees with the results of previous steady-state analyses.

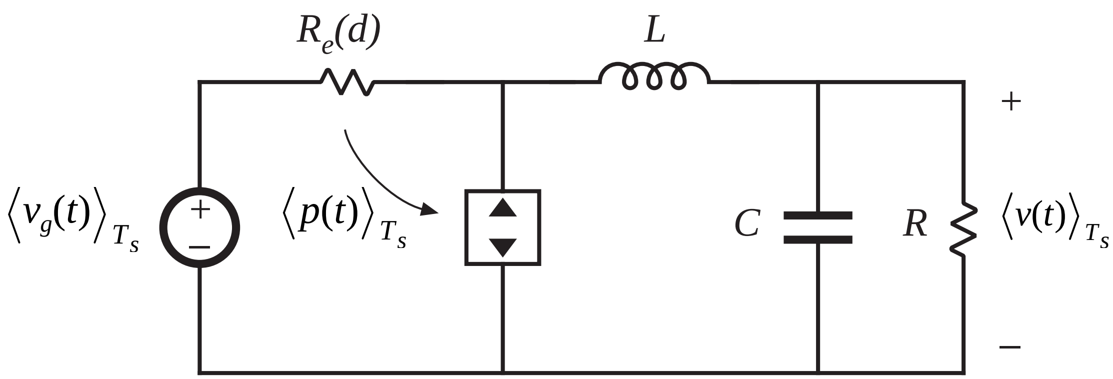
Averaged models of other DCM converters

- Determine averaged terminal waveforms of switch network
- In each case, averaged transistor waveforms obey Ohm's law, while averaged diode waveforms behave as dependent power source
- **Can simply replace transistor and diode with the averaged model** as follows:



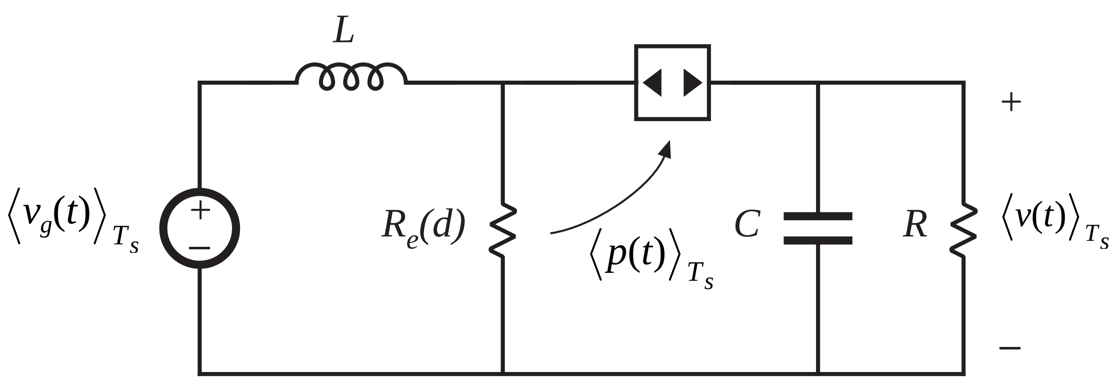
DCM buck, boost

Buck



$$R_e = \frac{2L}{d^2 T_s}$$

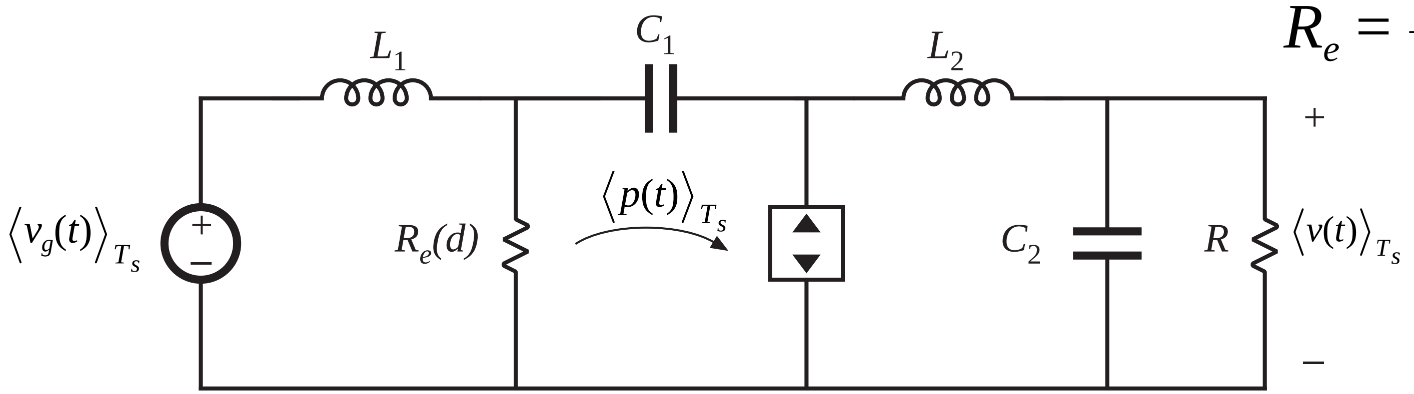
Boost



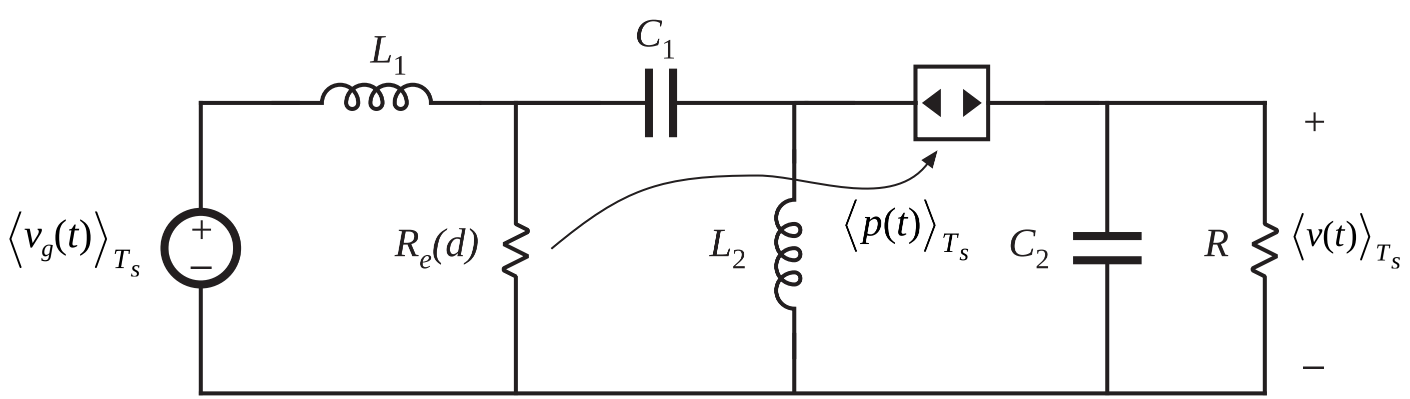
DCM Cuk, SEPIC

$$R_e = \frac{2\left(L_1 \| L_2\right)}{d^2 T_s}$$

Cuk



SEPIC

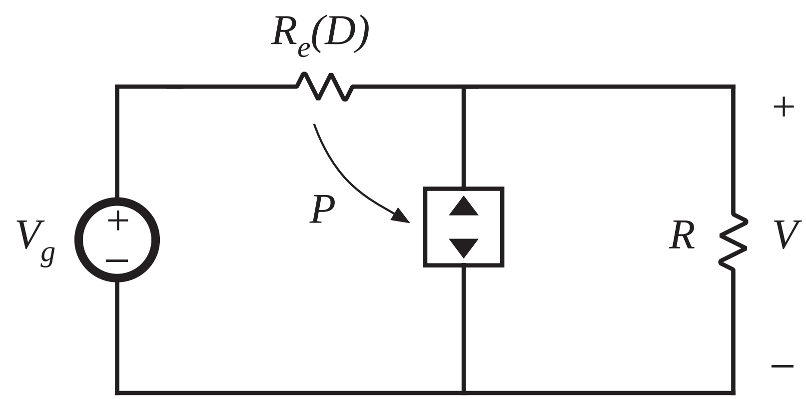


Steady-state solution: DCM buck, boost

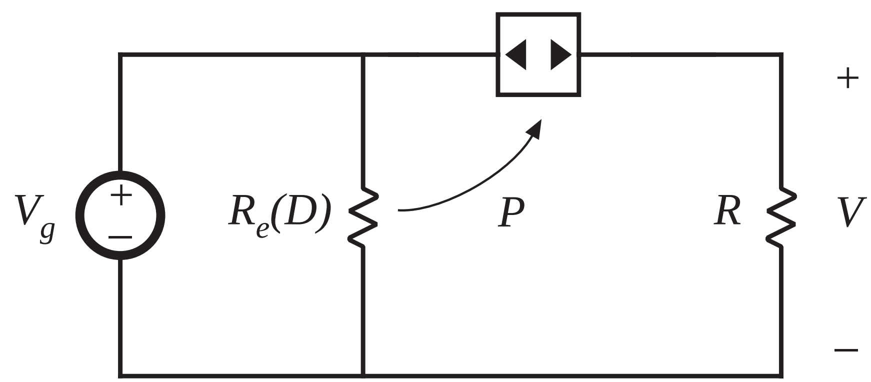
Let $L \rightarrow$ short circuit

$C \rightarrow$ open circuit

Buck



Boost



Steady-state solution of DCM/LFR models

Converter	M, CCM	M, DCM
Buck	D	$\dfrac{2}{1+\sqrt{1+4R_e/R}}$
Boost	$\dfrac{1}{1-D}$	$\dfrac{1+\sqrt{1+4R/R_e}}{2}$
Buck-boost, Cuk	$\dfrac{-D}{1-D}$	$-\sqrt{\dfrac{R}{R_e}}$
SEPIC	$\dfrac{D}{1-D}$	$\sqrt{\dfrac{R}{R_e}}$

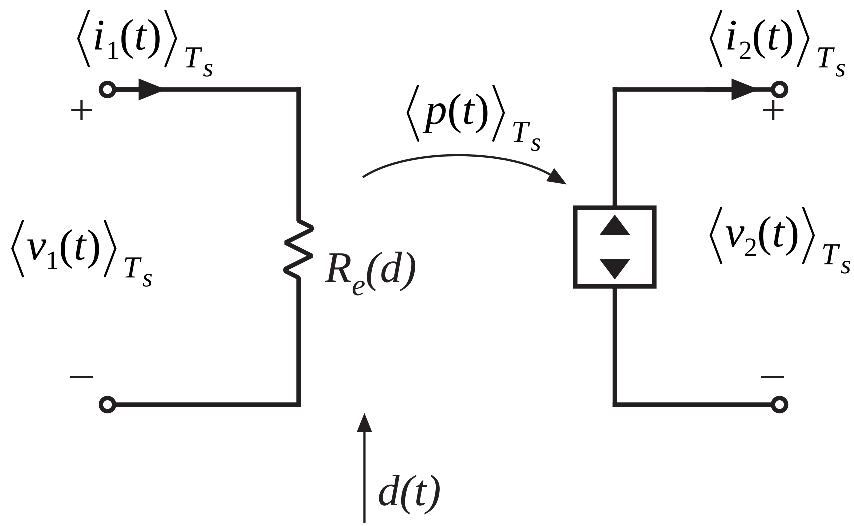
$I > I_{crit}$ for CCM

$I < I_{crit}$ for DCM

$$I_{crit} = \frac{1-D}{D}\,\frac{V_g}{R_e(D)}$$

Small-signal ac modeling of the DCM switch network

Large-signal averaged model



$$\langle i_1(t)\rangle_{T_s} = \frac{d_1^2(t)\,T_s}{2L}\,\langle v_1(t)\rangle_{T_s}$$

$$\langle i_2(t)\rangle_{T_s} = \frac{d_1^2(t)\,T_s}{2L}\,\frac{\langle v_1(t)\rangle_{T_s}^2}{\langle v_2(t)\rangle_{T_s}}$$

Perturb and linearize: let

$$d(t) = D + \hat{d}(t)$$

$$\langle v_1(t)\rangle_{T_s} = V_1 + \hat{v}_1(t)$$

$$\langle i_1(t)\rangle_{T_s} = I_1 + \hat{i}_1(t)$$

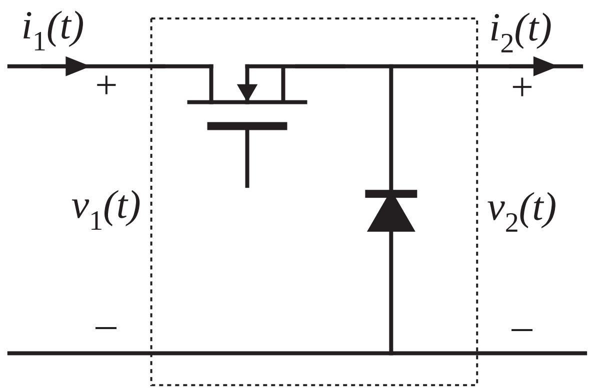
$$\langle v_2(t)\rangle_{T_s} = V_2 + \hat{v}_2(t)$$

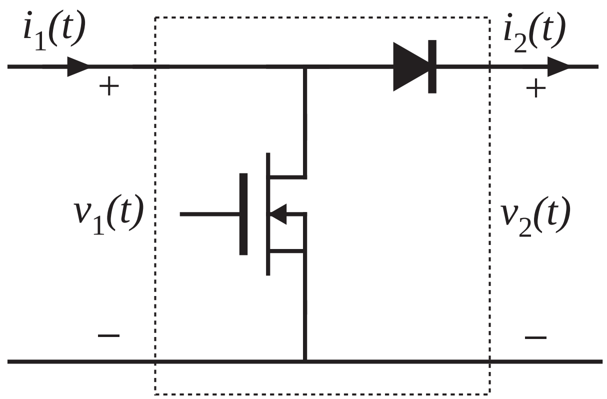
$$\langle i_2(t)\rangle_{T_s} = I_2 + \hat{i}_2(t)$$

$$\hat{i}_1 = \frac{\hat{v}_1}{r_1} + j_1\hat{d} + g_1\hat{v}_2$$

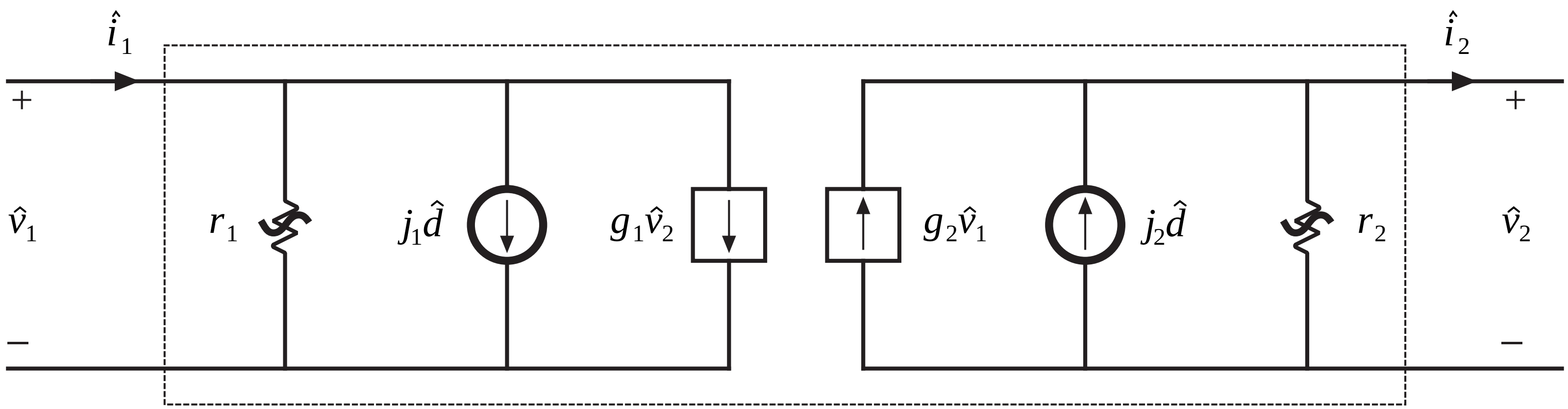
$$\hat{i}_2 = -\frac{\hat{v}_2}{r_2} + j_2\hat{d} + g_2\hat{v}_1$$

A more convenient way to model the buck and boost small-signal DCM switch networks

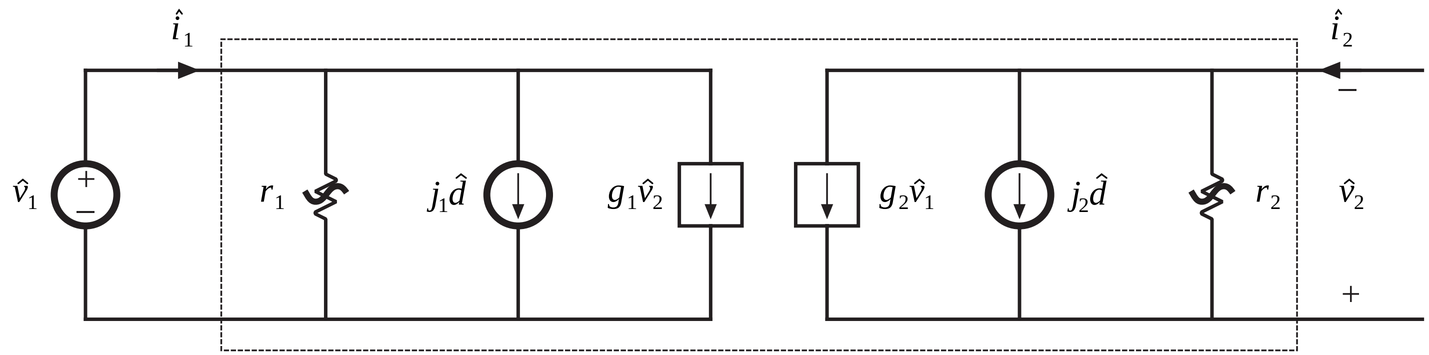




In any event, a small-signal two-port model is used, of the form



Small-signal DCM switch model parameters



Switch type	g_1	j_1	r_1	g_2	j_2	r_2
Buck,	$\frac{1}{R_e}$	$\frac{2(1-M)V_1}{DR_e}$	R_e	$\frac{2-M}{MR_e}$	$\frac{2(1-M)V_1}{DMR_e}$	M^2R_e
Boost,	$\frac{1}{(M-1)^2 R_e}$	$\frac{2MV_1}{D(M-1)R_e}$	$\frac{(M-1)^2}{M}R_e$	$\frac{2M-1}{(M-1)^2 R_e}$	$\frac{2V_1}{D(M-1)R_e}$	$(M-1)^2R_e$
Buck-boost,	0	$\frac{2V_1}{DR_e}$	R_e	$\frac{2M}{R_e}$	$\frac{2V_1}{DMR_e}$	M^2R_e

DCM small-signal transfer functions

- When expressed in terms of R, L, C, and M (not D), the small-signal transfer functions are the same in DCM as in CCM
- Hence, DCM boost and buck-boost converters exhibit two poles and one RHP zero in control-to-output transfer functions
- ***But***, value of L is small in DCM. Hence

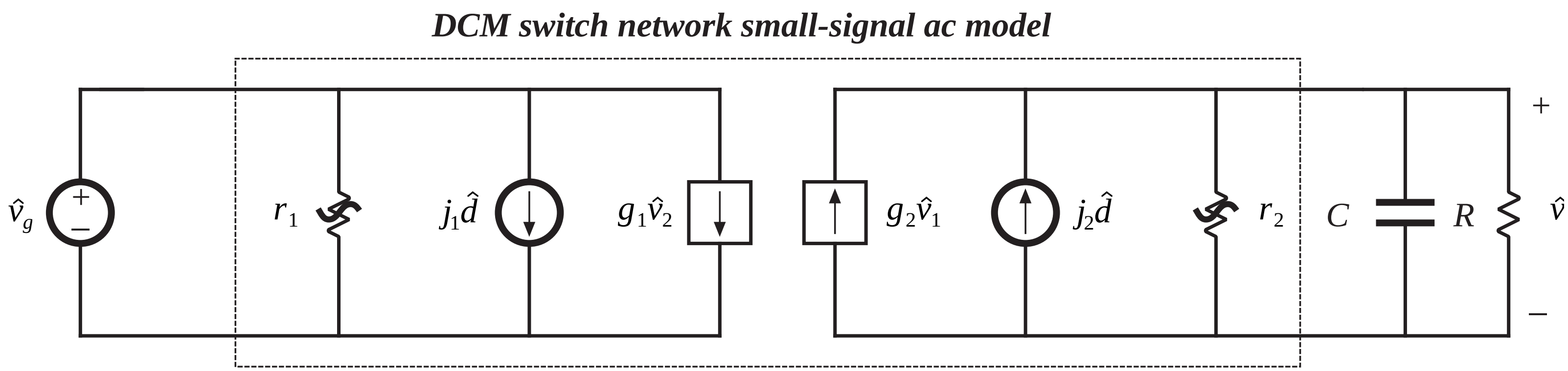
  RHP zero appears at high frequency, usually greater than switching frequency

  Pole due to inductor dynamics appears at high frequency, near to or greater than switching frequency

  So DCM buck, boost, and buck-boost converters exhibit essentially a single-pole response
- A simple approximation: let $L \to 0$

The simple approximation $L \to 0$

Buck, boost, and buck-boost converter models all reduce to



Transfer functions

control-to-output

$$G_{vd}(s) = \left.\frac{\hat{v}}{\hat{d}}\right|_{\hat{v}_g = 0} = \frac{G_{d0}}{1 + \frac{s}{\omega_p}}$$

with

$$G_{d0} = j_2 \left(R \,\|\, r_2\right)$$

$$\omega_p = \frac{1}{\left(R \,\|\, r_2\right) C}$$

$$G_{g0} = g_2 \left(R \,\|\, r_2\right) = M$$

line-to-output

$$G_{vg}(s) = \left.\frac{\hat{v}}{\hat{v}_g}\right|_{\hat{d} = 0} = \frac{G_{g0}}{1 + \frac{s}{\omega_p}}$$

Transfer function salient features

Converter	G_{d0}	G_{g0}	ω_p
Buck	$\frac{2V}{D}\frac{1-M}{2-M}$	M	$\frac{2-M}{(1-M)RC}$
Boost	$\frac{2V}{D}\frac{M-1}{2M-1}$	M	$\frac{2M-1}{(M-1)RC}$
Buck-boost	$\frac{V}{D}$	M	$\frac{2}{RC}$

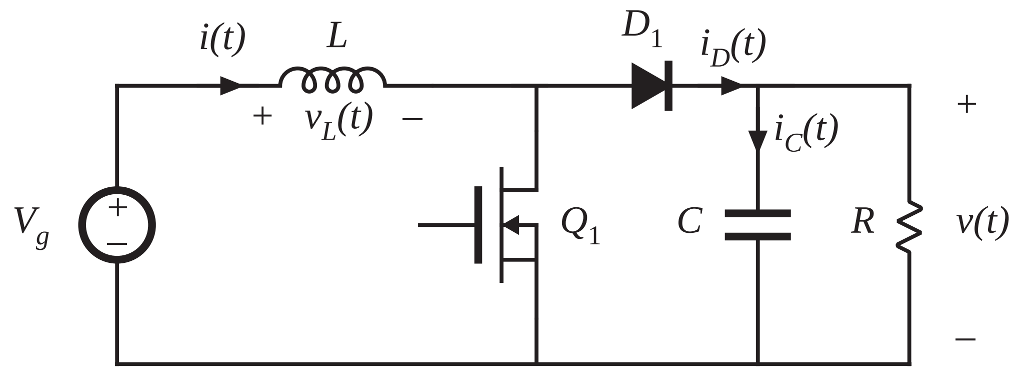
DCM boost example
Control-to-output transfer function $G_{vd}(s)$

$R = 12\ \Omega$

$L = 5\ \mu H$

$C = 470\ \mu F$

$f_s = 100\ kHz$



The output voltage is regulated to be $V = 36$ V. It is desired to determine $G_{vd}(s)$ at the operating point where the load current is $I = 3$ A and the dc input voltage is $V_g = 24$ V.

Evaluate simple model parameters

$$P = I\left(V - V_g\right) = \left(3\text{ A}\right)\left(36\text{ V} - 24\text{ V}\right) = 36\text{ W}$$

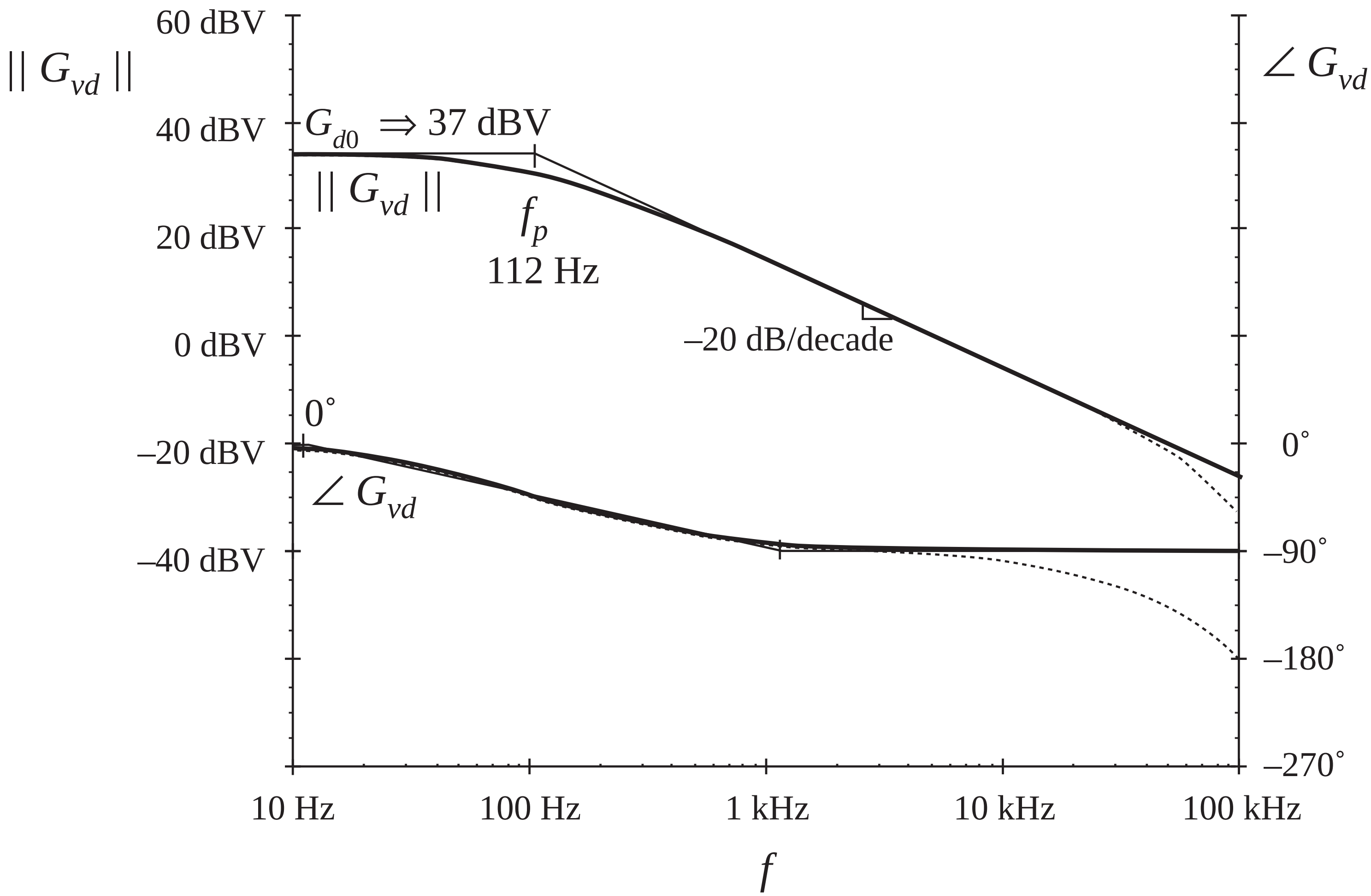
$$R_e = \frac{V_g^2}{P} = \frac{(24\text{ V})^2}{36\text{ W}} = 16\ \Omega$$

$$D = \sqrt{\frac{2L}{R_e T_s}} = \sqrt{\frac{2(5\ \mu\text{H})}{(16\ \Omega)(10\ \mu\text{s})}} = 0.25$$

$$G_{d0} = \frac{2V}{D}\ \frac{M-1}{2M-1} = \frac{2(36\text{ V})}{(0.25)}\ \frac{\left(\frac{(36\text{ V})}{(24\text{ V})} - 1\right)}{\left(2\ \frac{(36\text{ V})}{(24\text{ V})} - 1\right)} = 72\text{ V} \Rightarrow 37\text{ dBV}$$

$$f_p = \frac{\omega_p}{2\pi} = \frac{2M-1}{2\pi\,(M-1)RC} = \frac{\left(2\ \frac{(36\text{ V})}{(24\text{ V})} - 1\right)}{2\pi\left(\frac{(36\text{ V})}{(24\text{ V})} - 1\right)(12\ \Omega)(470\ \mu\text{F})} = 112\text{ Hz}$$

Control-to-output transfer function, boost example



A Note on High-Frequency Predictions of the Averaged Switch Model

- Observed high-frequency response due to inductor dynamics
- Averaged-switch model derivation used:

$$\left\langle v_L \right\rangle_{T_s} = 0$$

which is consistent with the fact that in DCM the inductor current starts from zero and ends at zero in each switching cycle, even in transients

- However, high-frequency dynamics due to the inductor indicates that the AC voltage across the inductor in the small-signal model is *not* zero
- *Model predictions at high frequencies are not quite correct*
- Corrected averaged models that *include the inductor* in the averaged switch model have recently been described

See References: [Sun et. al. PESC'99], [Ben-Yaakov et.al. PESC'94]

Combined CCM/DCM
Average Switch Model

Objective: a general large-signal averaged-switch model

- Valid in CCM and DCM
- 5 terminals:

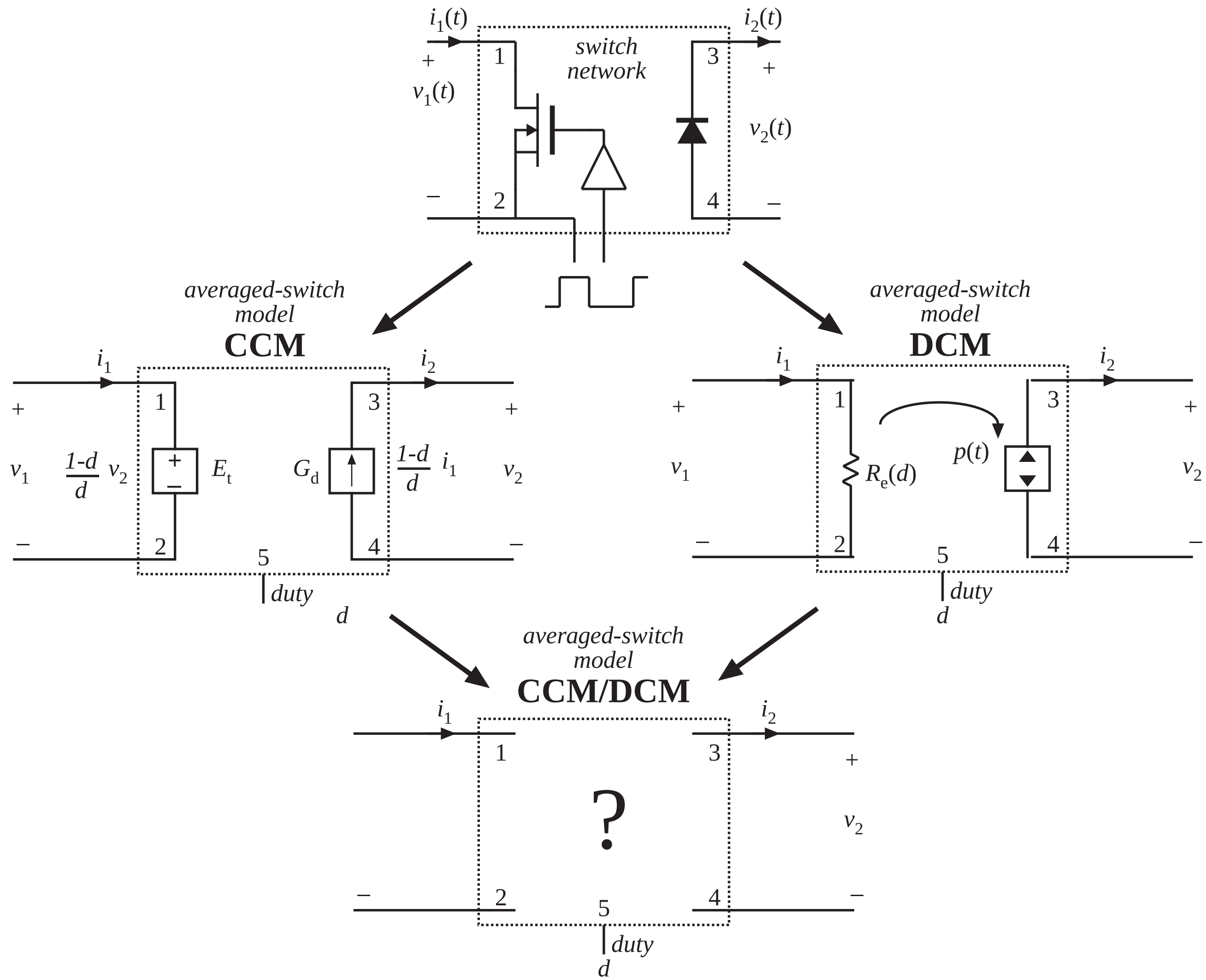
    transistor port (2 terminals)

    diode port (2 terminals)

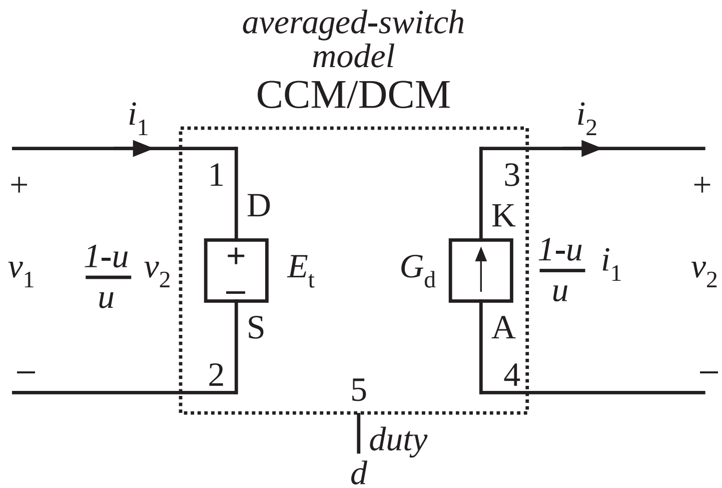
    duty ratio input (1 terminal)

- DCM/CCM boundary resolved within the model, based only on the terminal voltages/currents of the model
- Spice compatible

Combined CCM/DCM Average Switch Model



Combined CCM/DCM Average Switch Model



$$u = \begin{cases} d, & CCM \\ \dfrac{d^2}{d^2 + 2Lf_s \dfrac{i_1}{v_2}}, & DCM \end{cases}$$

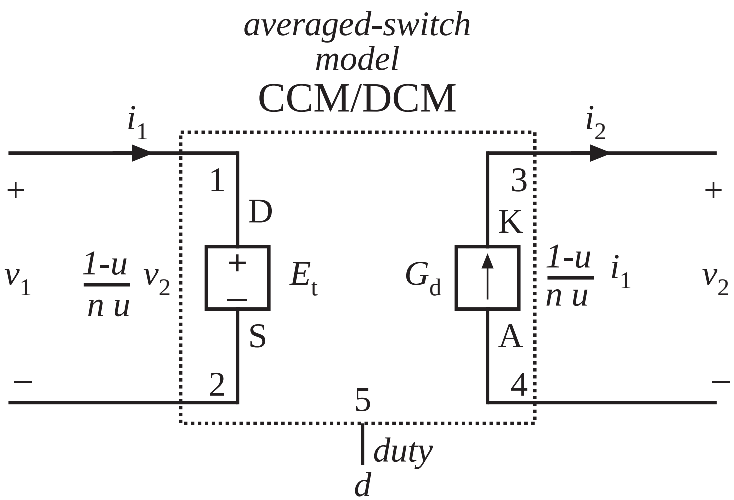
CCM/DCM boundary: $$u = MAX\left(d, \quad \frac{d^2}{d^2 + 2Lf_s \dfrac{i_1}{v_2}} \right)$$

u = equivalent switch duty ratio

CCM / DCM Averaged-Switch Model
PSpice Implementation: ccm-dcm1

```
***************************************************************************************
* MODEL: ccm-dcm1
* Application: two-switch PWM converters, CCM or DCM
* Limitations: ideal switches, no transformer
***************************************************************************************
* Parameters:
*     L=equivalent inductance (relevant for DCM)
*     fs=switching frequency
***************************************************************************************
* Nodes: (same as in ccm1)
***************************************************************************************
.subckt ccm-dcm1 1  2  3  4  5 params: L=1 fs=1E6
Et 1 2 value={(1-v(u))*v(3,4)/v(u)}
Gd 4 3 value={(1-v(u))*i(Et)/v(u)}
Ga 0 a value={MAX(i(Et),0)}
Va a b
Rdummy b 0 10
Eu u 0 table {MAX(v(5), v(5)*v(5)/(v(5)*v(5)+2*L*fs*i(Va)/v(3,4)))} (0 0) (1 1)
.ends
***************************************************************************************
```

Combined CCM/DCM Average Switch Model With Isolation Transformer



$$u = \begin{cases} d, & CCM \\ \dfrac{d^2}{d^2 + 2nLf_s \dfrac{i_1}{v_2}}, & DCM \end{cases}$$

CCM/DCM boundary: $$u = MAX\left(d, \quad \frac{d^2}{d^2 + 2nLf_s \dfrac{i_1}{v_2}} \right)$$

u = equivalent switch duty ratio

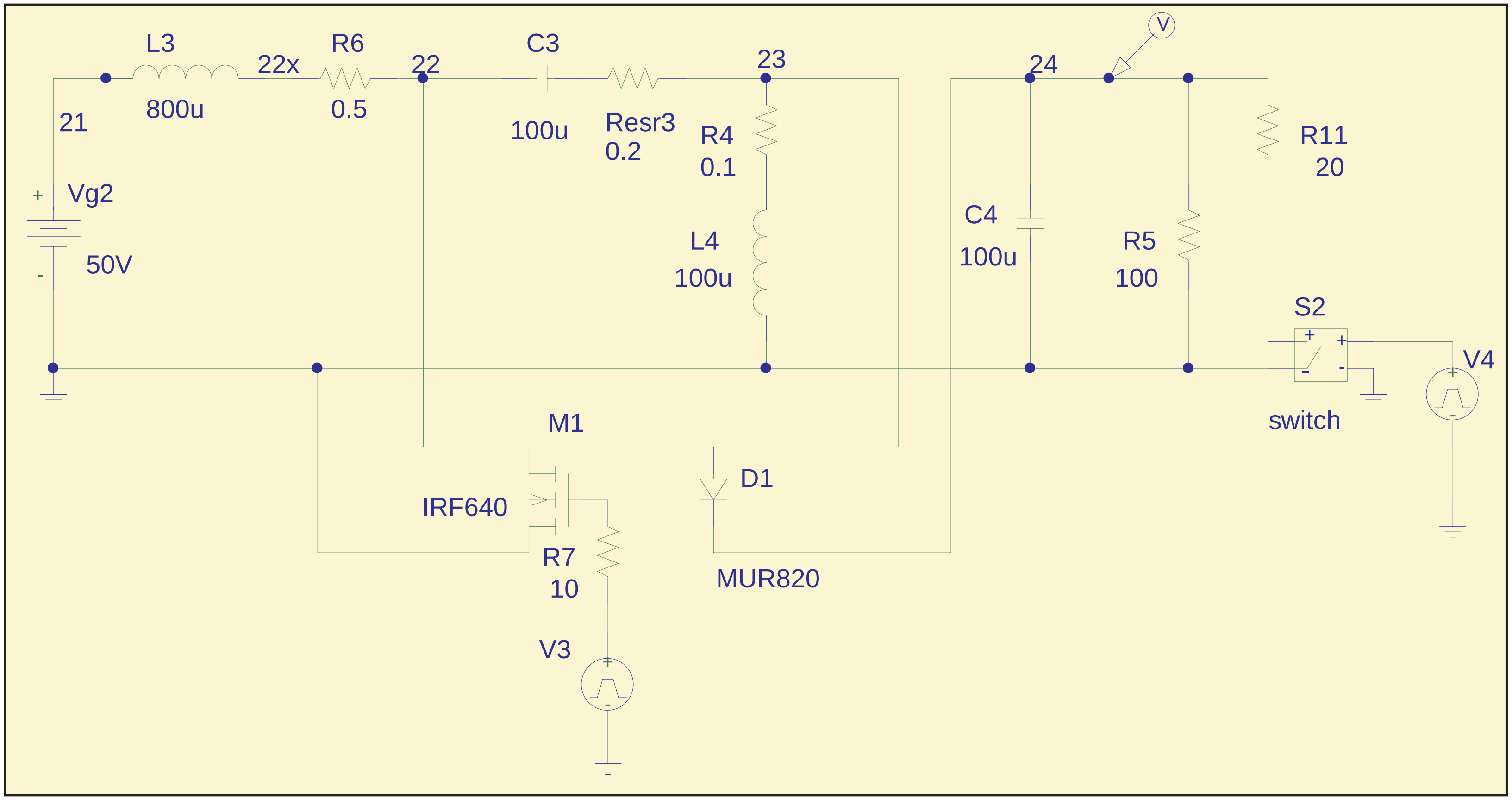
CCM / DCM Averaged-Switch Model
PSpice Implementation: ccm-dcm2

```
* MODEL: ccm-dcm2
* Application: two-switch PWM converters, CCM or DCM with (possibly) transformer
* Limitations: ideal switches, no transformer
****************************************************************************************
* Parameters:
*      L=equivalent inductance (relevant for DCM), referred to primary
*      fs=switching frequency
*      n=transformer turns ratio 1:n (primary:secondary)
****************************************************************************************
* Nodes: (same as in ccm1)
****************************************************************************************
.subckt ccm-dcm2 1  2  3  4  5 params: L=1 fs=1E6 n=1
Et 1 2 value={(1-v(u))*v(3,4)/v(u)/n}
Gd 4 3 value={(1-v(u))*i(Et)/v(u)/n}
Ga 0 a value={MAX(i(Et),0)}
Va a b
Rdummy b 0 10
Eu u 0 table {MAX(v(5), v(5)*v(5)/(v(5)*v(5)+2*L*n*fs*i(Va)/v(3,4)))} (0 0) (1 1)
.ends
****************************************************************************************
```

CCM/DCM Model Applications

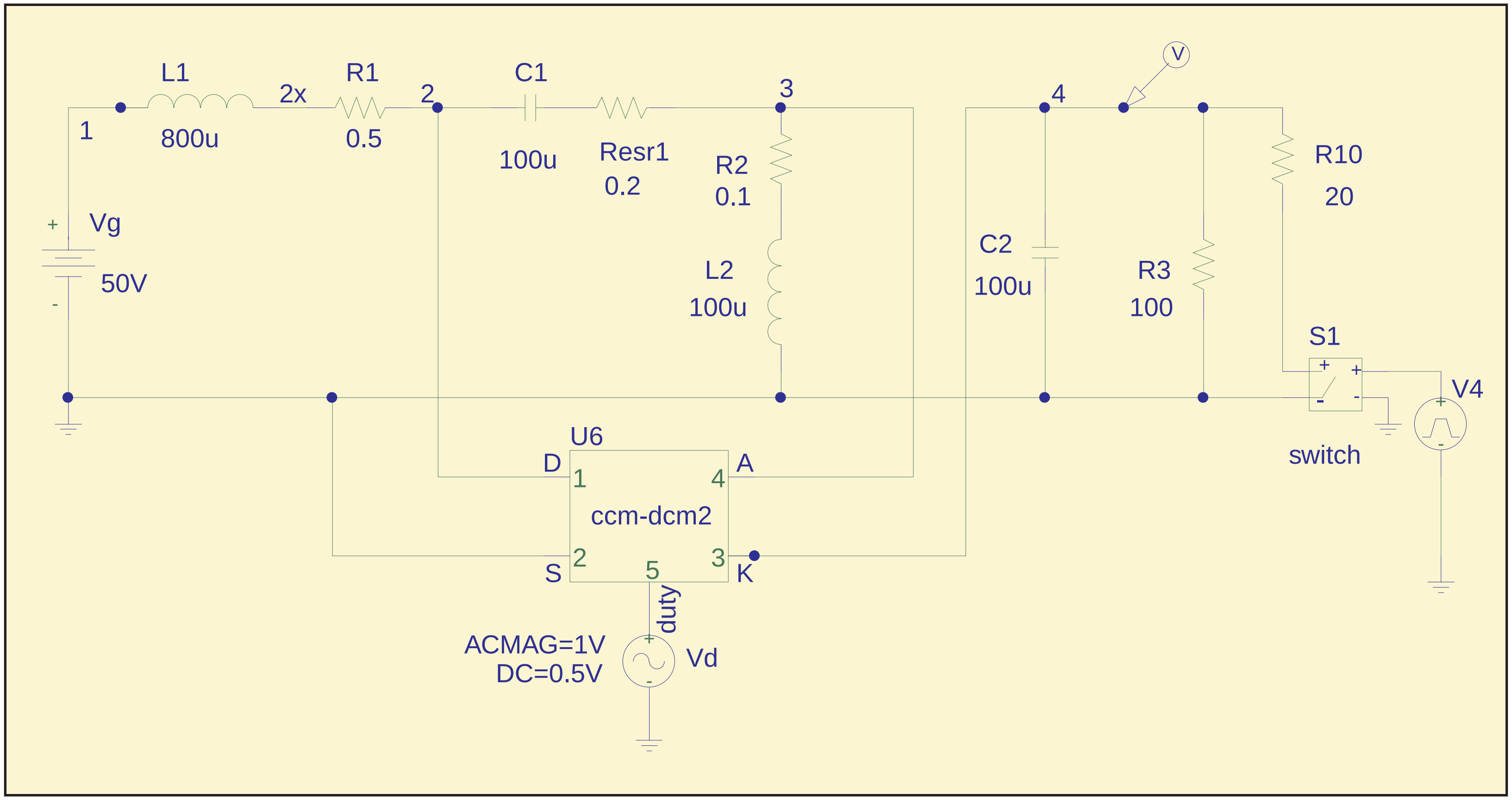
- **ccm-dcm1** (for non-isolated converters) and **ccm-dcm2** (for converters that may include isolation transformer) are general, large-signal averaged-switch models (PSpice subcircuits) valid for both CCM and DCM
- Can be applied to DC, AC, or Transient simulation of any two-switch PWM converter
- Limitations: ideal switches, no losses are modeled, but the model can be refined further to include conduction losses
- **Application examples:**
  - Comparison of Transient simulation results in a Sepic converter example using:
    - (1) switching circuit model
    - (2) ccm-dcm2 averaged switch model
  - AC simulation results for a flyback converter operating in CCM or DCM

Sepic converter example: switching circuit model



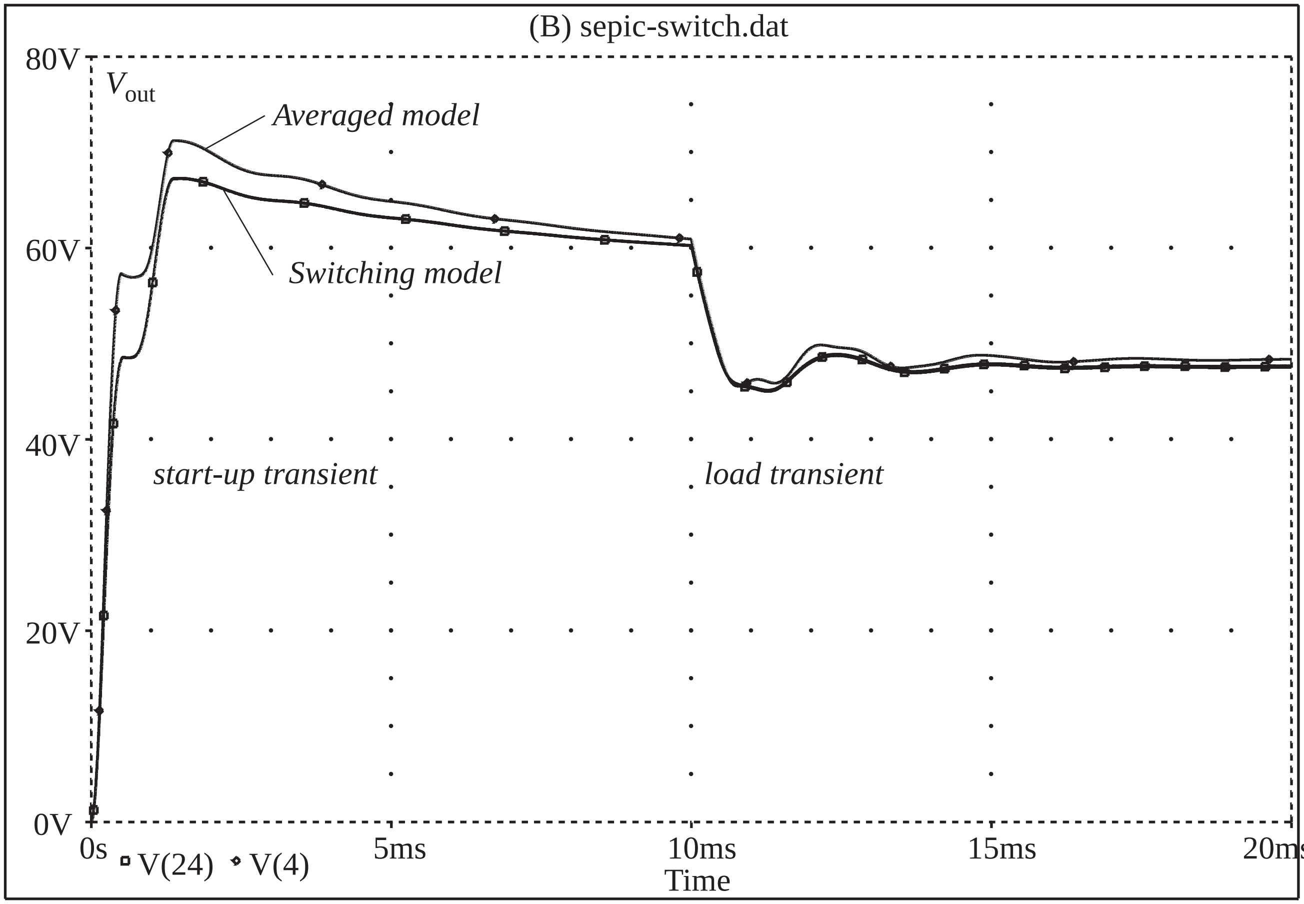
Switching frequency 100kHz, duty ratio D=0.5

Sepic converter example: averaged model using ccm-dcm2



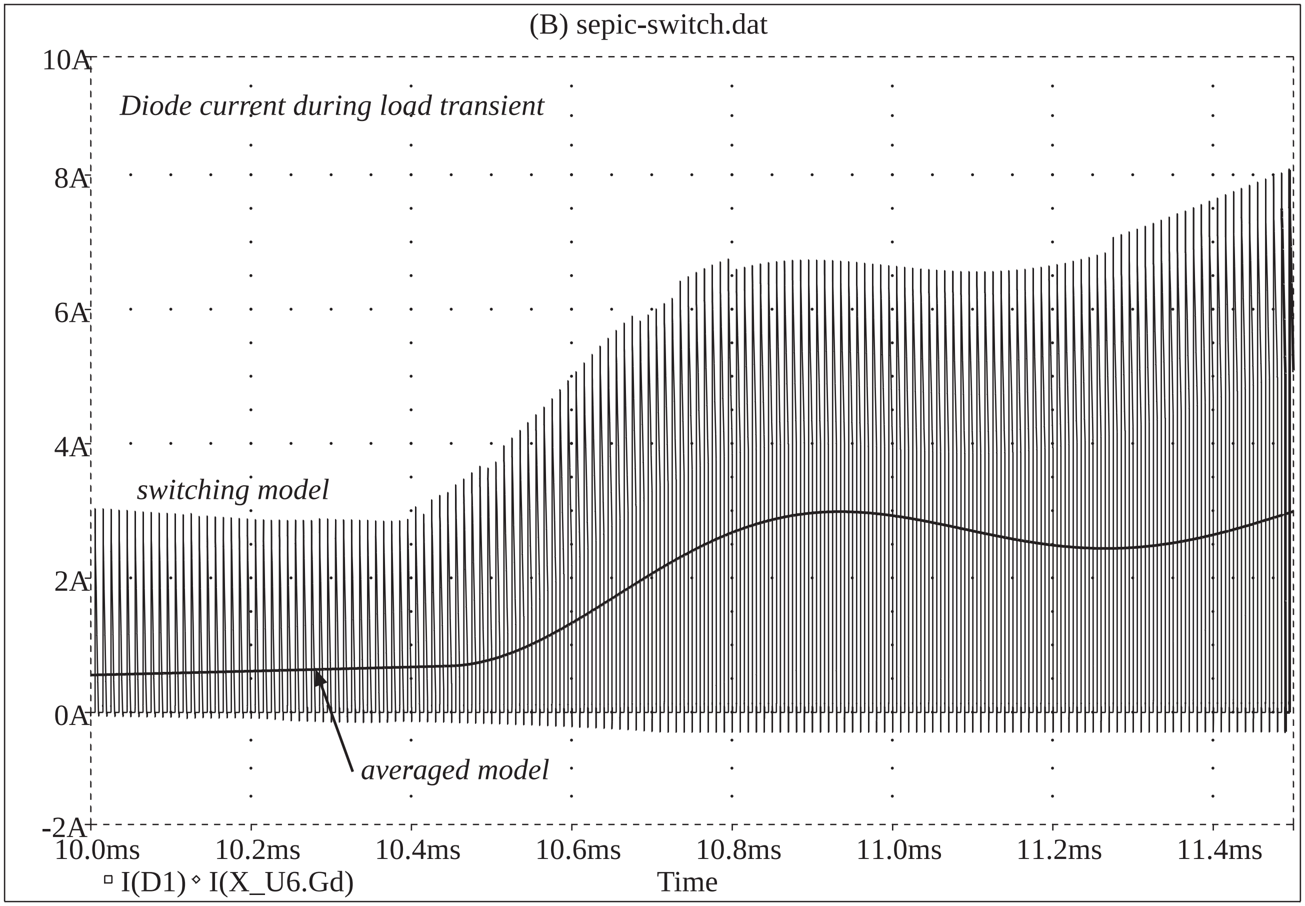
Exactly the same PSpice circuit, except the MOSFET M1 and the diode D1 replaced by the **ccm-dcm2** subcircuit, and pulsating gate drive V3 replaced by a duty-ratio voltage source Vd

Sepic converter example: averaged vs. switching model



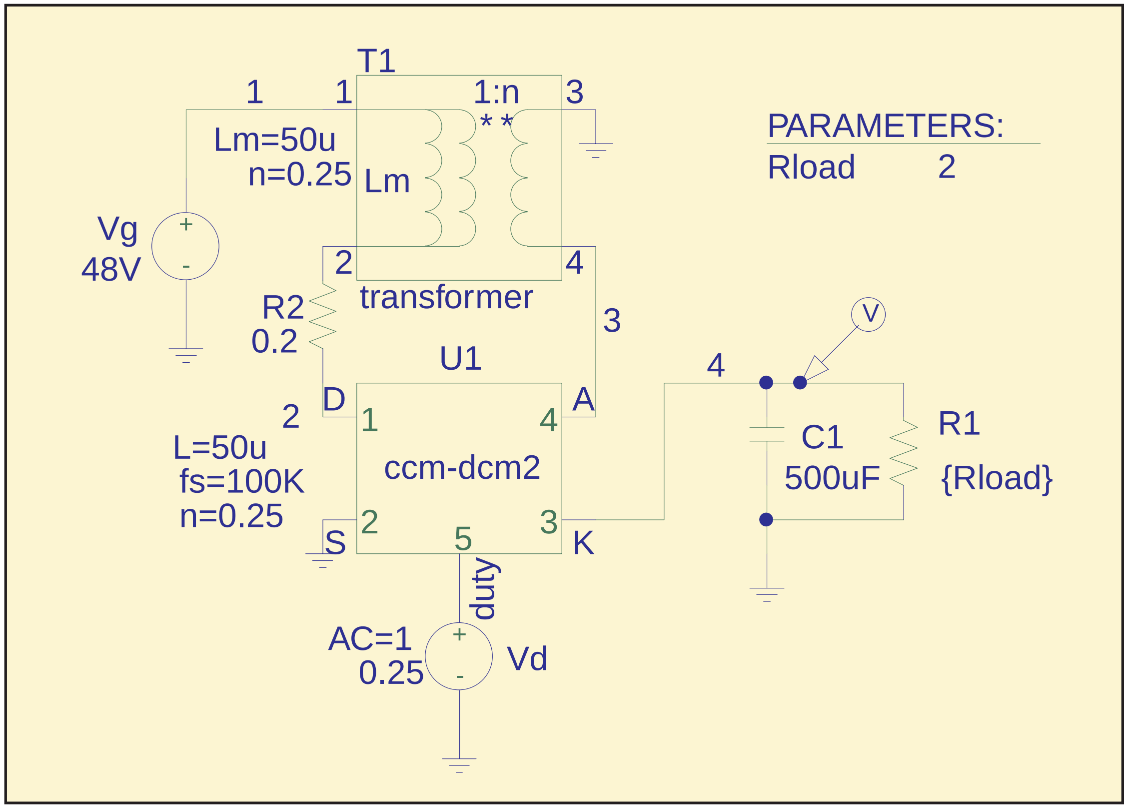
Start-up and load transient response

Sepic converter example: averaged vs. switching model



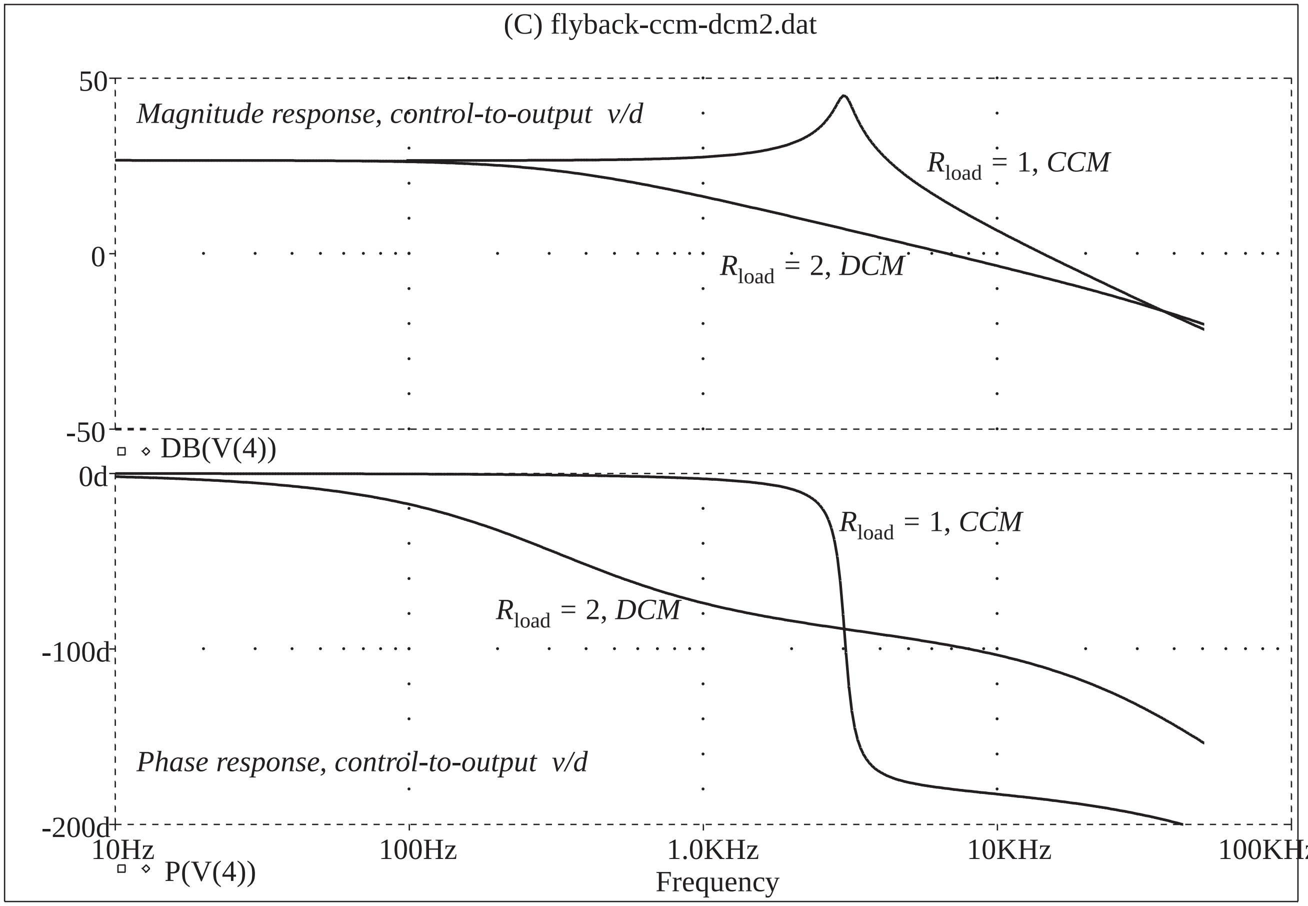
Details of the diode current waveform around the load transient

Flyback converter example
using ccm-dcm2 averaged-switch model



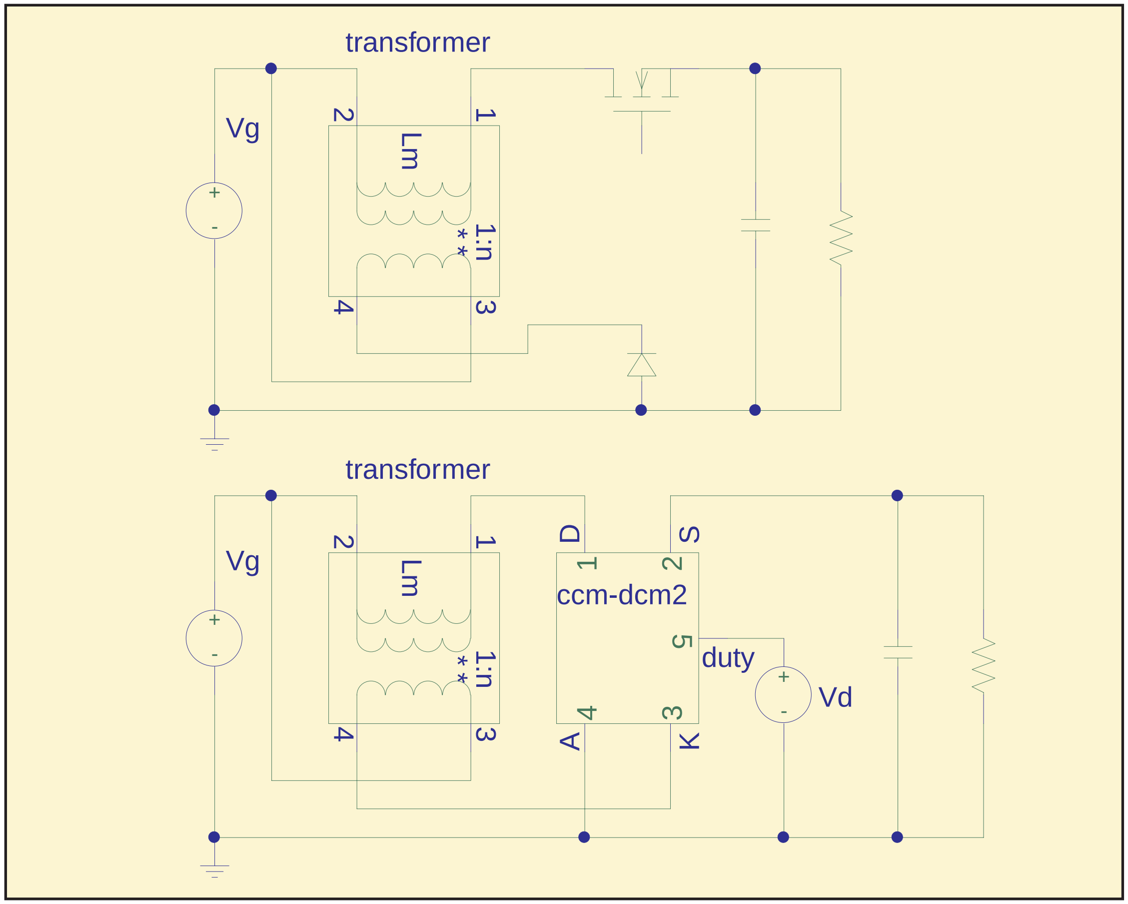
CCM for Rload=1 Ohm, DCM for Rload=2 Ohm

Flyback converter example using ccm-dcm2 averaged-switch model



Frequency responses generated by PSpice AC analyses

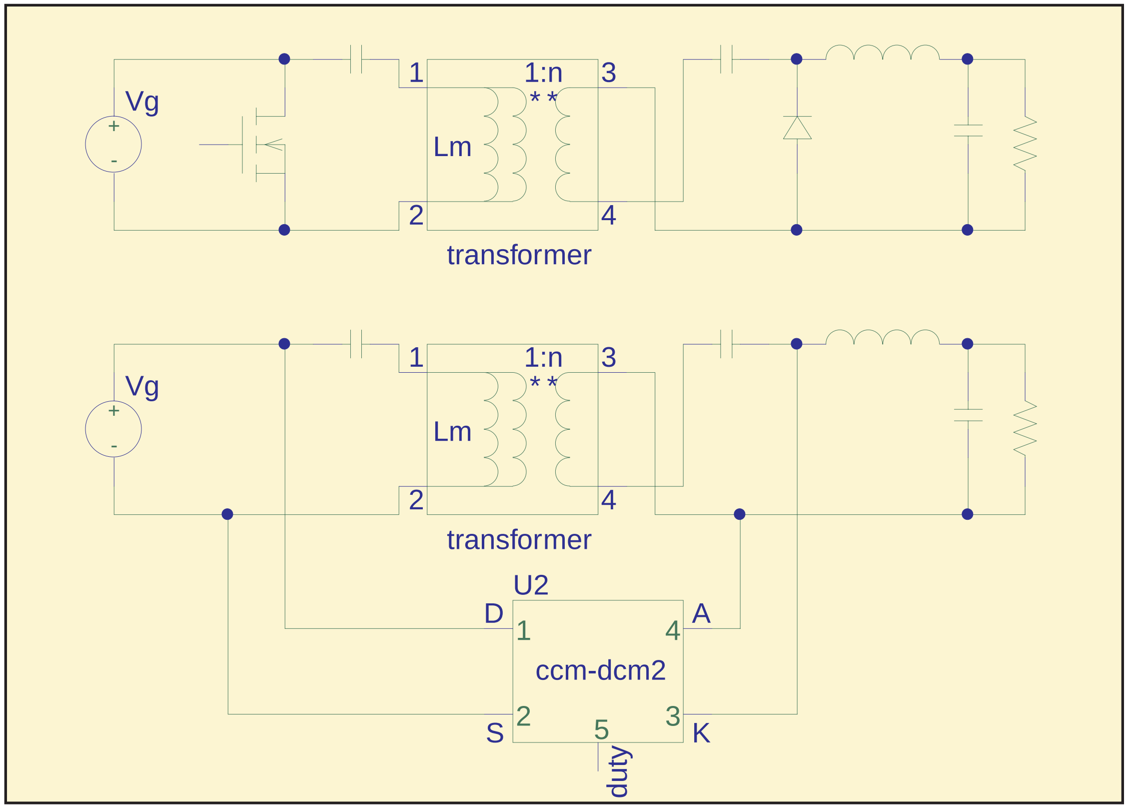
Other Converter Examples



Watkins-Johnson converter

Pspice averaged circuit model using **ccm-dcm2** averaged-switch subcircuit

Other Converter Examples



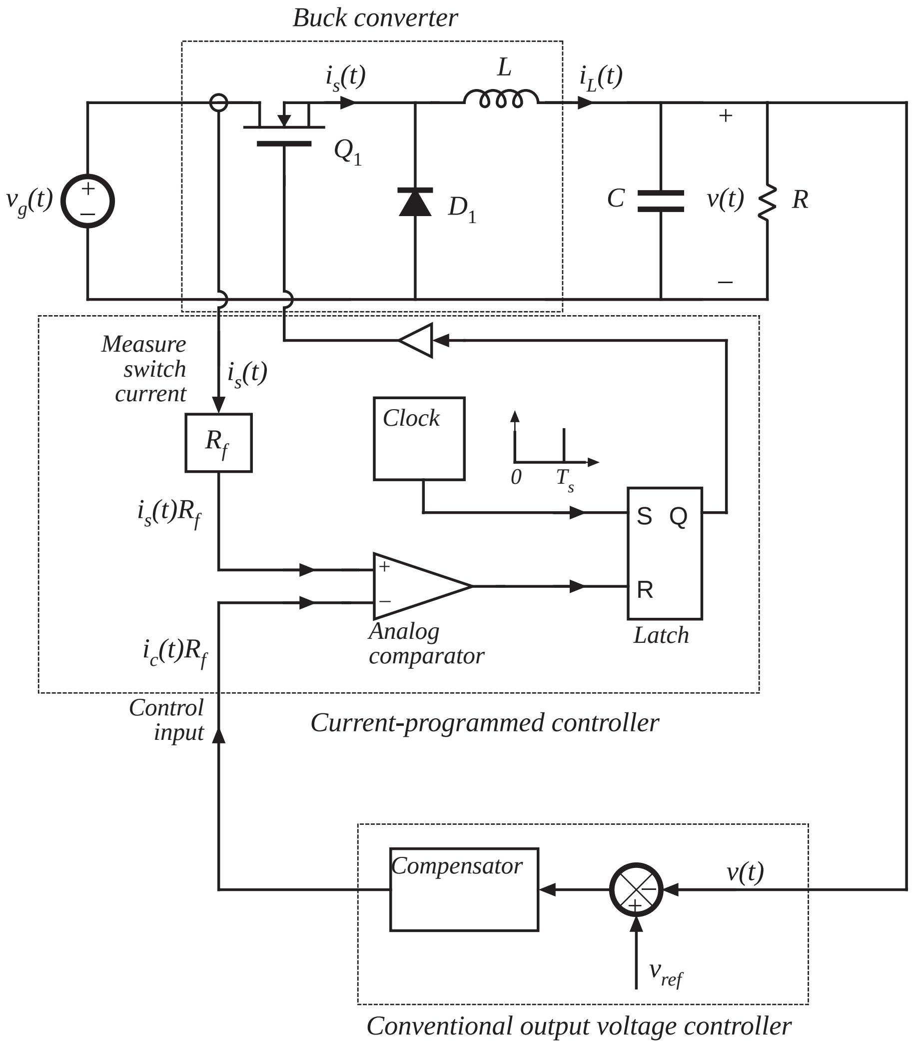
Cuk converter with isolation transformer

PSpice averaged circuit model using **ccm-dcm2** averaged-switch subcircuit

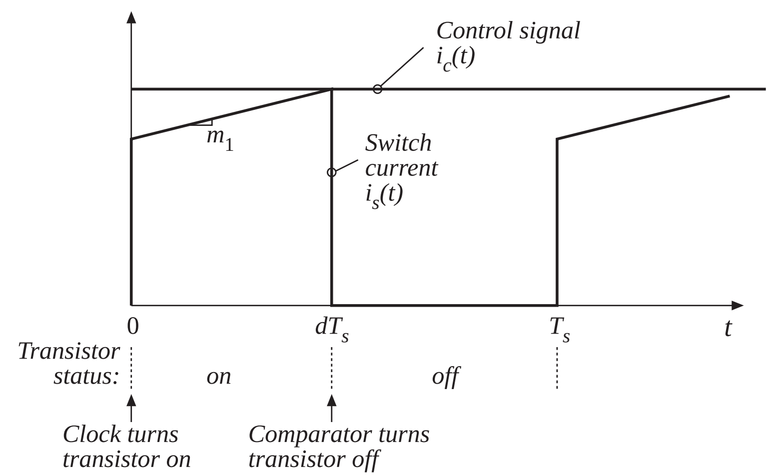
4. Averaged modeling of PWM converters with current-programmed mode (CPM) control

- **Averaged switch model for current-programmed mode (CPM) in CCM**
- **Steady-state and simple AC model in CCM**
- **Averaged switch model for CPM in DCM**
- **Steady-state and small-signal AC model in DCM**
- **Large-signal averaged CCM/DCM model for current-mode controller**
- **PSpice implementation of the averaged CPM controller model**
- **Application examples**
  - Buck converter with current-programmed mode controller

Current-programmed control



The peak transistor current replaces the duty cycle as the converter control input.



A simple approximation

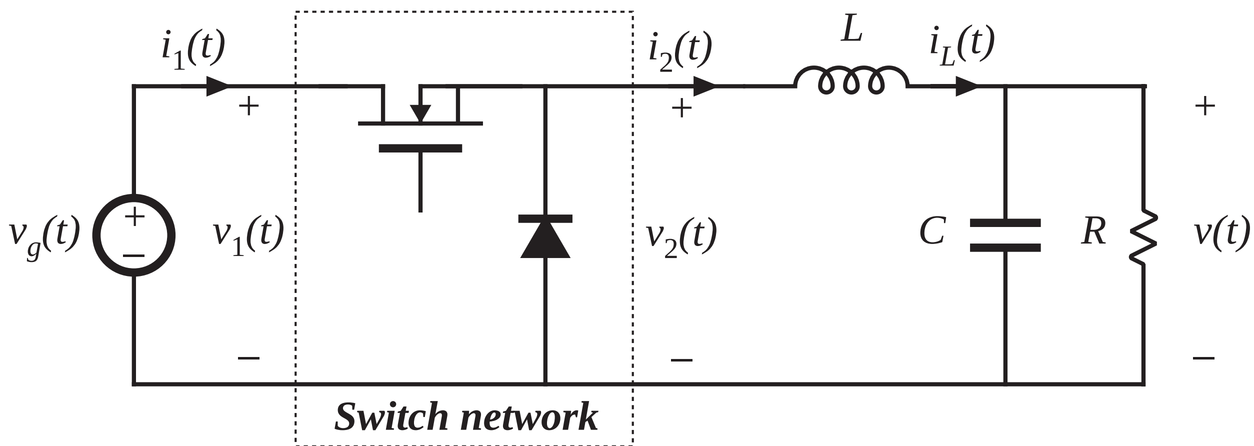
$$\left\langle i_L(t) \right\rangle_{T_s} = i_c(t)$$

- Neglects switching ripple and artificial ramp (slope compensation)
- Yields physical insight and simple first-order model
- Accurate when converter operates well into CCM (so that switching ripple is small) and when the magnitude of the artificial ramp is not too large
- Well-accepted by practicing engineers
- Resulting small-signal relation:

$$\hat{i}_L(s) \approx \hat{i}_c(s)$$

Averaged switch modeling
with the simple approximation

Buck converter example



Averaged terminal waveforms, CCM:

$$\left\langle v_2(t) \right\rangle_{T_s} = d(t) \left\langle v_1(t) \right\rangle_{T_s}$$

$$\left\langle i_1(t) \right\rangle_{T_s} = d(t) \left\langle i_2(t) \right\rangle_{T_s}$$

The simple approximation:

$$\left\langle i_2(t) \right\rangle_{T_s} \approx \left\langle i_c(t) \right\rangle_{T_s}$$

CPM averaged switch equations

$$\left\langle v_2(t)\right\rangle_{T_s} = d(t)\left\langle v_1(t)\right\rangle_{T_s}$$

$$\left\langle i_1(t)\right\rangle_{T_s} = d(t)\left\langle i_2(t)\right\rangle_{T_s}$$

$$\left\langle i_2(t)\right\rangle_{T_s} \approx \left\langle i_c(t)\right\rangle_{T_s}$$

Eliminate duty cycle:

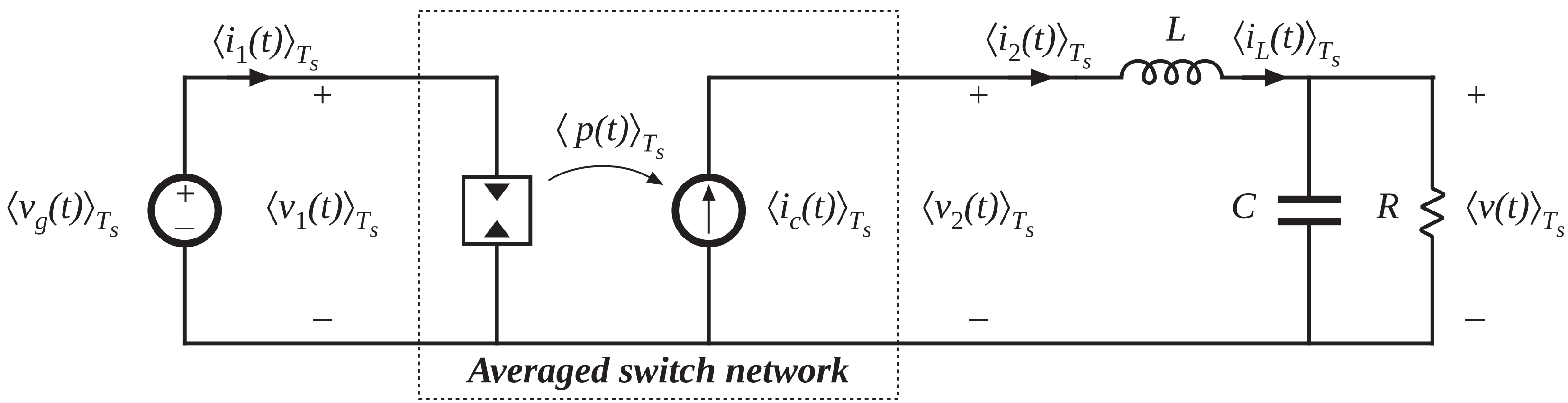
$$\left\langle i_1(t)\right\rangle_{T_s} = d(t)\left\langle i_c(t)\right\rangle_{T_s} = \frac{\left\langle v_2(t)\right\rangle_{T_s}}{\left\langle v_1(t)\right\rangle_{T_s}}\left\langle i_c(t)\right\rangle_{T_s}$$

$$\left\langle i_1(t)\right\rangle_{T_s}\left\langle v_1(t)\right\rangle_{T_s} = \left\langle i_c(t)\right\rangle_{T_s}\left\langle v_2(t)\right\rangle_{T_s} = \left\langle p(t)\right\rangle_{T_s}$$

So:

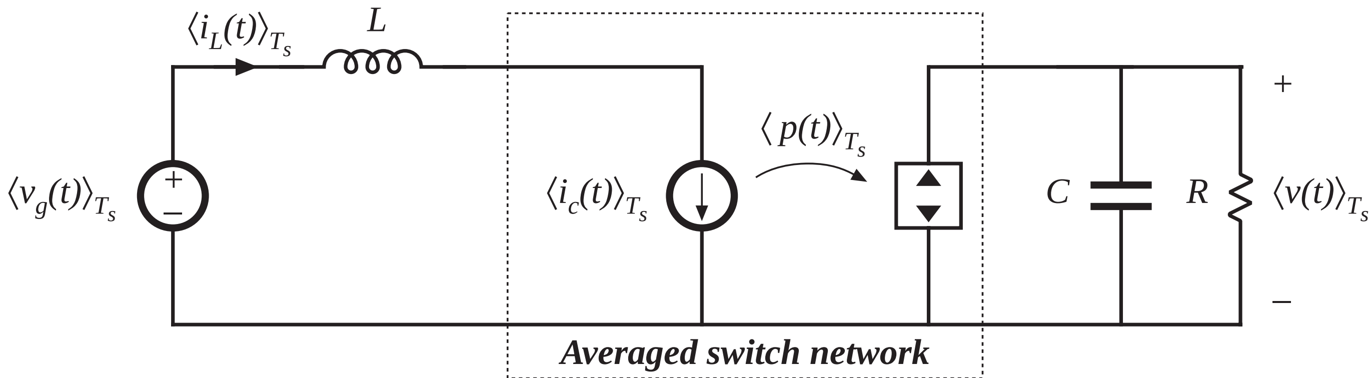
- Output port is a current source
- Input port is a dependent power sink

CPM averaged switch model

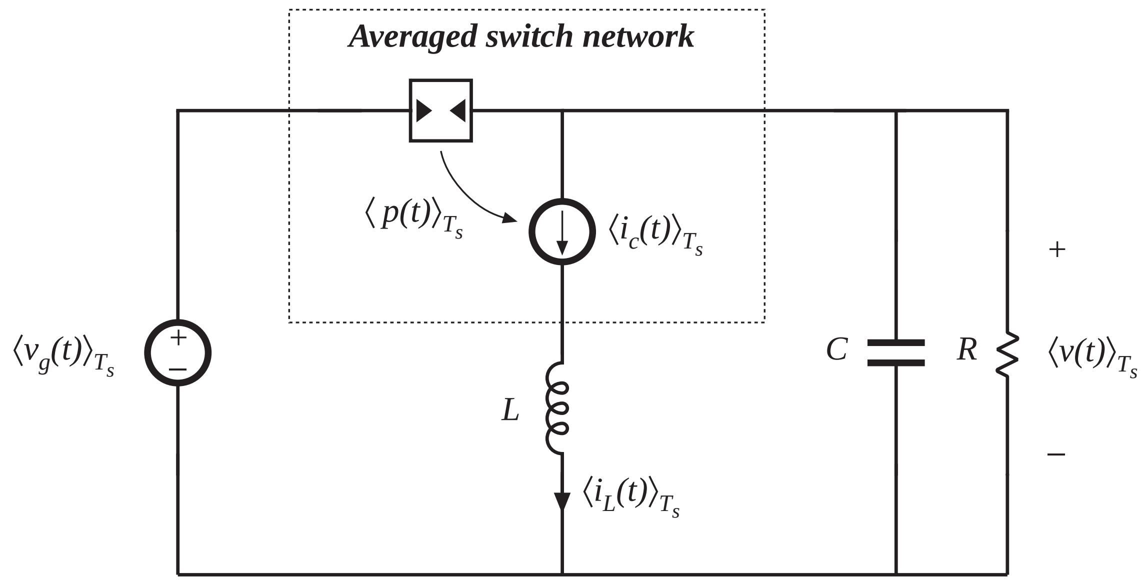


Results for other converters

Boost



Buck-boost



Perturbation and linearization
to construct small-signal model, CCM

Let

$$\left\langle v_1(t) \right\rangle_{T_s} = V_1 + \hat{v}_1(t)$$

$$\left\langle i_1(t) \right\rangle_{T_s} = I_1 + \hat{i}_1(t)$$

$$\left\langle v_2(t) \right\rangle_{T_s} = V_2 + \hat{v}_2(t)$$

$$\left\langle i_2(t) \right\rangle_{T_s} = I_2 + \hat{i}_2(t)$$

$$\left\langle i_c(t) \right\rangle_{T_s} = I_c + \hat{i}_c(t)$$

Resulting input port equation:

$$\left(V_1 + \hat{v}_1(t) \right) \left(I_1 + \hat{i}_1(t) \right) = \left(I_c + \hat{i}_c(t) \right) \left(V_2 + \hat{v}_2(t) \right)$$

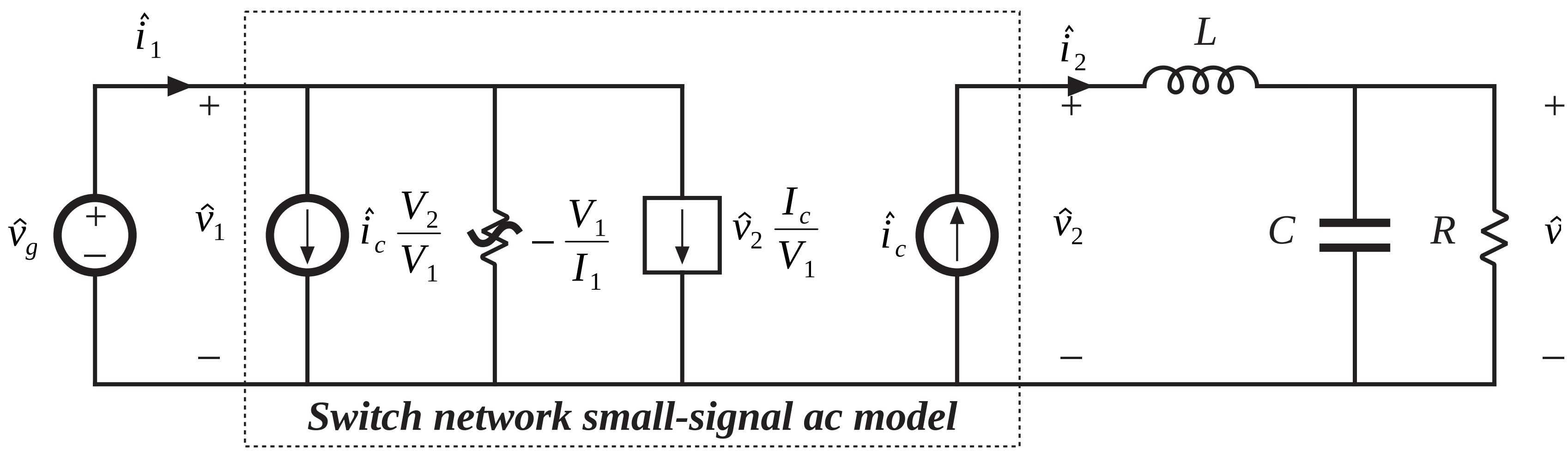
Small-signal result:

$$\hat{i}_1(t) = \hat{i}_c(t) \frac{V_2}{V_1} + \hat{v}_2(t) \frac{I_c}{V_1} - \hat{v}_1(t) \frac{I_1}{V_1}$$

Output port equation:

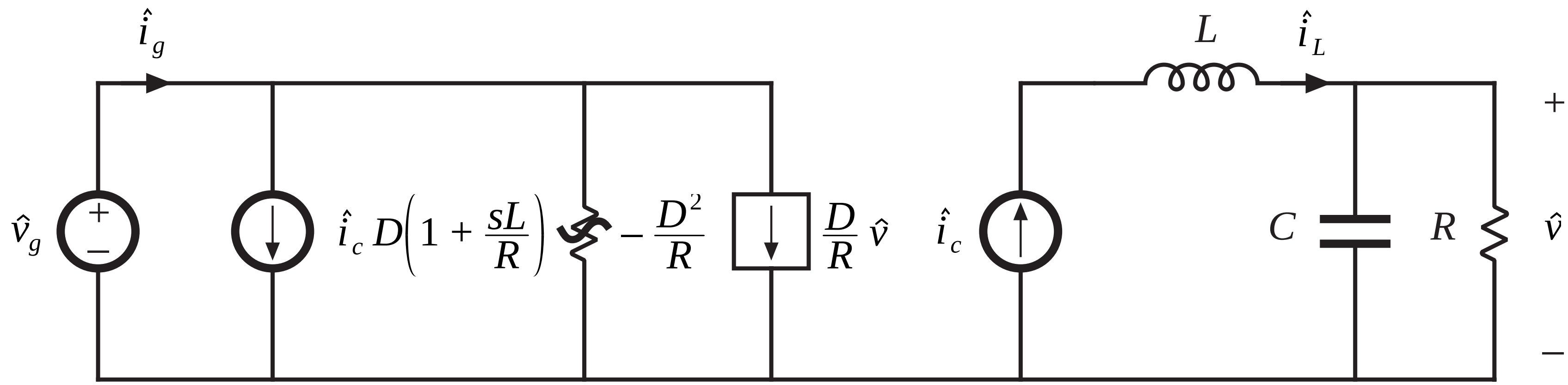
$$\hat{i}_2 = \hat{i}_c$$

Resulting small-signal model
Buck example



$$\hat{i}_1(t) = \hat{i}_c(t)\,\frac{V_2}{V_1} + \hat{v}_2(t)\,\frac{I_c}{V_1} - \hat{v}_1(t)\,\frac{I_1}{V_1}$$

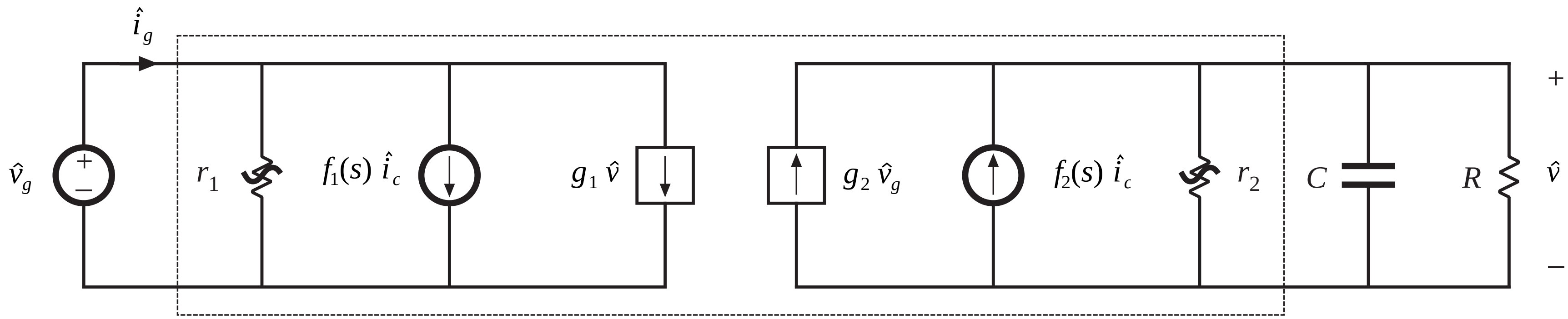
Predicted transfer functions of the CPM buck converter



$$G_{vc}(s) = \left.\frac{\hat{v}(s)}{\hat{i}_c(s)}\right|_{\hat{v}_g = 0} = \left(R \parallel \frac{1}{sC}\right)$$

$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{i}_c = 0} = 0$$

Table of results
basic converters



Converter	g_1	f_1	r_1	g_2	f_2	r_2
Buck	$\frac{D}{R}$	$D\left(1+\frac{sL}{R}\right)$	$-\frac{R}{D^2}$	0	1	∞
Boost	0	1	∞	$\frac{1}{D'R}$	$D'\left(1-\frac{sL}{D'^2R}\right)$	R
Buck-boost	$-\frac{D}{R}$	$D\left(1+\frac{sL}{D'R}\right)$	$-\frac{D'R}{D^2}$	$-\frac{D^2}{D'R}$	$-D'\left(1-\frac{sDL}{D'^2R}\right)$	$\frac{R}{D}$

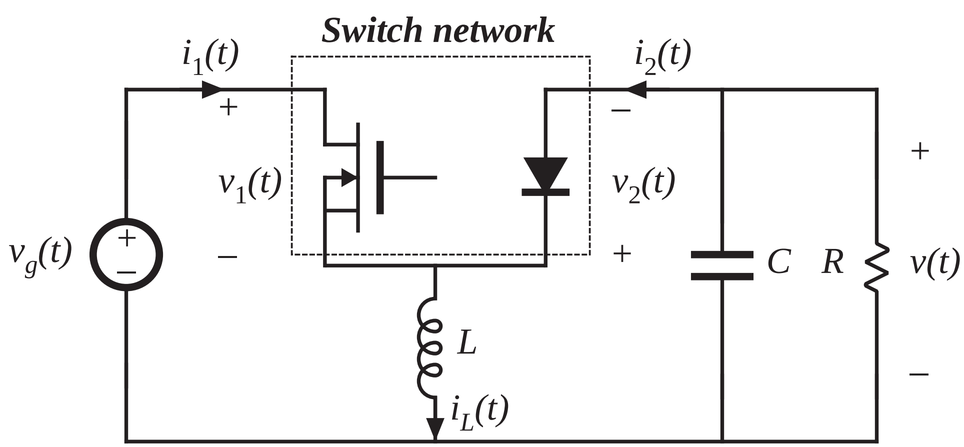
Discontinuous conduction mode
in current-programmed converters

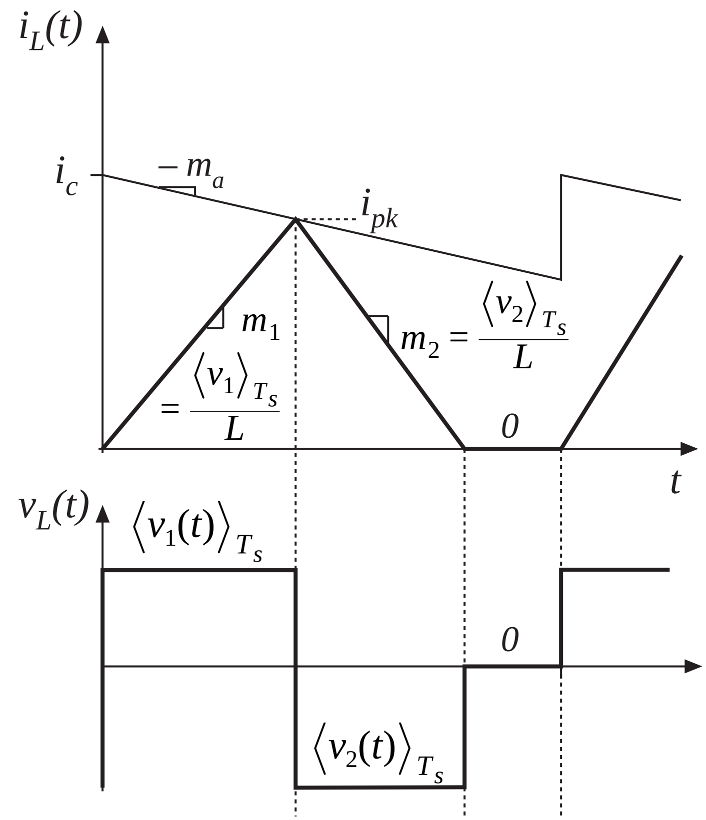
- Again, use averaged switch modeling approach
- Result: simply replace

  Transistor by power sink

  Diode by power source
- Inductor dynamics appear at high frequency, near to or greater than the switching frequency
- Small-signal transfer functions contain a single low frequency pole
- DCM CPM boost and buck-boost are stable without artificial ramp
- DCM CPM buck without artificial ramp is stable for $D < 2/3$. A small artificial ramp $m_a \geq 0.086 m_2$ leads to stability for all D.

DCM CPM buck-boost example





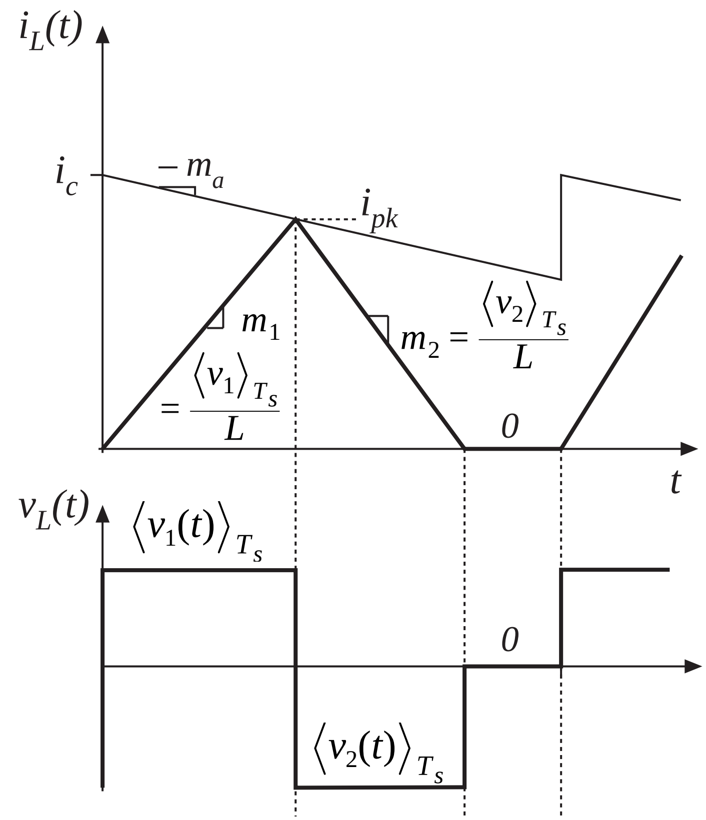
Analysis

$$i_{pk} = m_1 d_1 T_s$$

$$m_1 = \frac{\left\langle v_1(t) \right\rangle_{T_s}}{L}$$

$$i_c = i_{pk} + m_a d_1 T_s$$
$$= \left(m_1 + m_a\right) d_1 T_s$$

$$d_1(t) = \frac{i_c(t)}{\left(m_1 + m_a\right) T_s}$$



Averaged switch input port equation

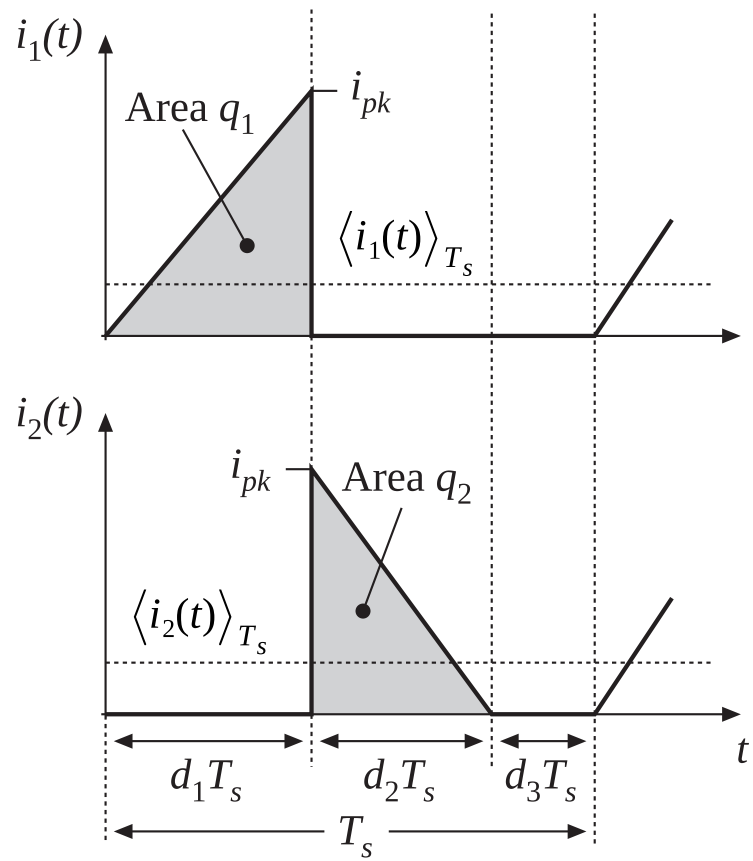
$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} i_1(\tau)\mathrm{d}\tau = \frac{q_1}{T_s}$$

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{1}{2}\, i_{pk}(t) d_1(t)$$

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{1}{2}\, m_1 d_1^2(t) T_s$$

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{\frac{1}{2} L i_c^2 f_s}{\left\langle v_1(t) \right\rangle_{T_s} \left(1 + \frac{m_a}{m_1}\right)^2}$$

$$\left\langle i_1(t) \right\rangle_{T_s} \left\langle v_1(t) \right\rangle_{T_s} = \frac{\frac{1}{2} L i_c^2 f_s}{\left(1 + \frac{m_a}{m_1}\right)^2} = \left\langle p(t) \right\rangle_{T_s}$$



Discussion: switch network input port

- Averaged transistor waveforms obey a power sink characteristic
- During first subinterval, energy is transferred from input voltage source, through transistor, to inductor, equal to

$$W = \frac{1}{2} L i_{pk}^2$$

This energy transfer process accounts for power flow equal to

$$\left\langle p(t) \right\rangle_{T_s} = W f_s = \frac{1}{2} L i_{pk}^2 f_s$$

which is equal to the power sink expression of the previous slide.

Averaged switch output port equation

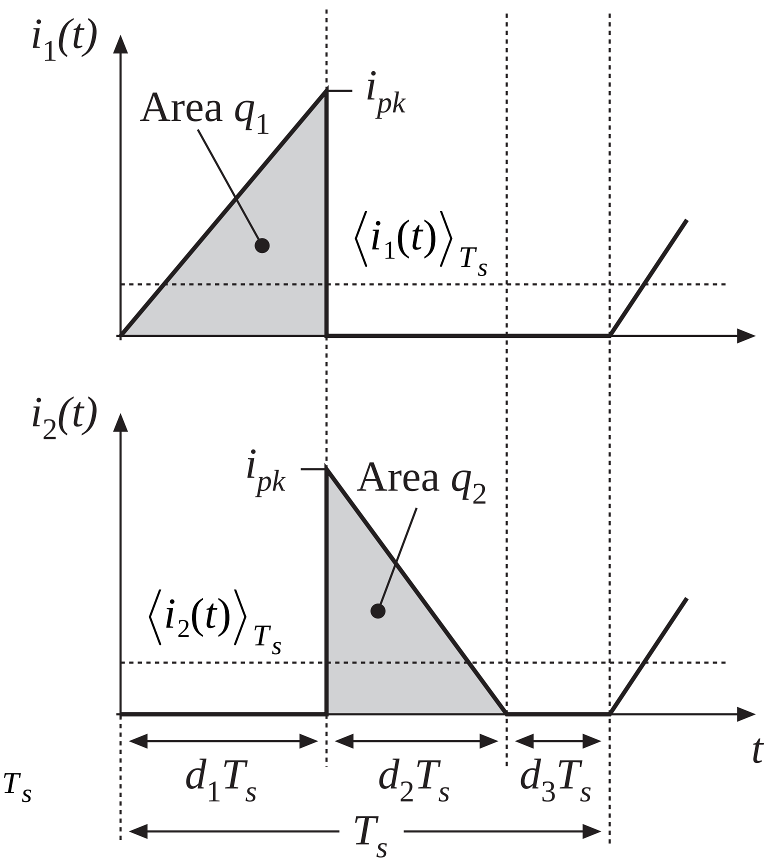
$$\left\langle i_2(t) \right\rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} i_2(\tau) d\tau = \frac{q_2}{T_s}$$

$$q_2 = \tfrac{1}{2}\, i_{pk} d_2 T_s$$

$$d_2(t) = d_1(t) \frac{\left\langle v_1(t) \right\rangle_{T_s}}{\left\langle v_2(t) \right\rangle_{T_s}}$$

$$\left\langle i_2(t) \right\rangle_{T_s} = \frac{\left\langle p(t) \right\rangle_{T_s}}{\left\langle v_2(t) \right\rangle_{T_s}}$$

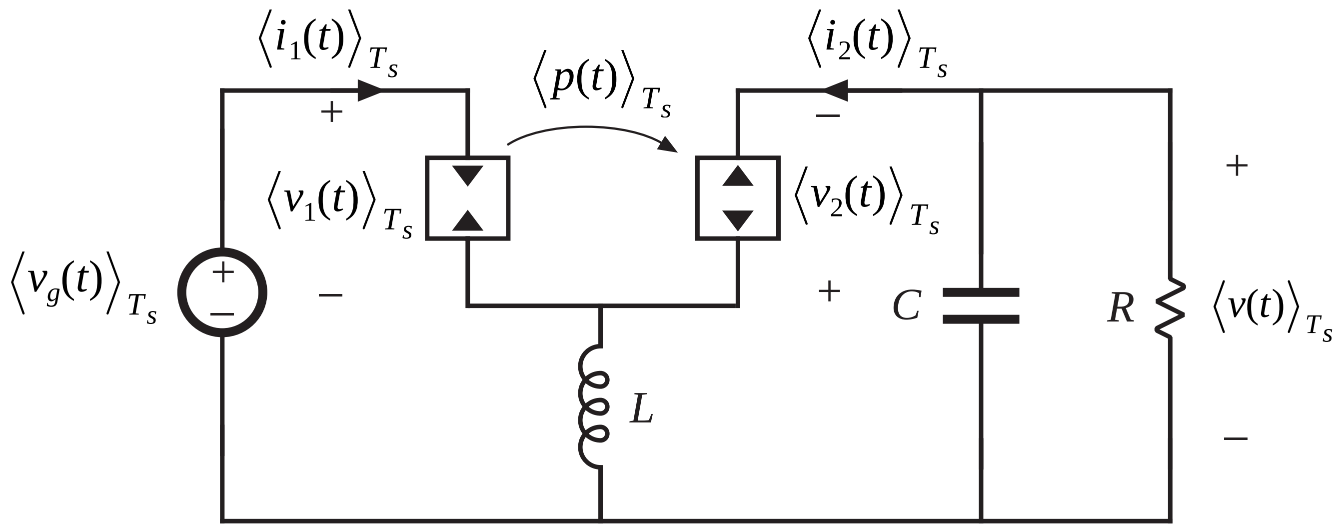
$$\left\langle i_2(t) \right\rangle_{T_s} \left\langle v_2(t) \right\rangle_{T_s} = \frac{\frac{1}{2} L i_c^2(t) f_s}{\left(1 + \frac{m_a}{m_1}\right)^2} = \left\langle p(t) \right\rangle_{T_s}$$



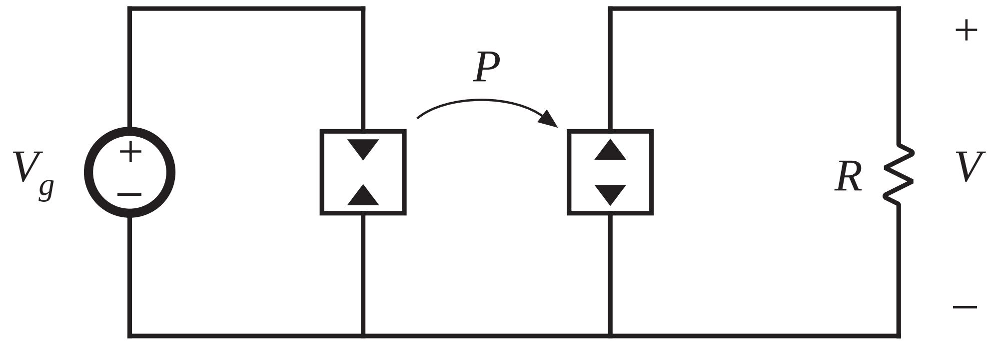
Discussion: switch network output port

- Averaged diode waveforms obey a power sink characteristic
- During second subinterval, all stored energy in inductor is transferred, through diode, to load
- Hence, in averaged model, diode becomes a power source, having value equal to the power consumed by the transistor power sink element

Averaged equivalent circuit



Steady state model: DCM CPM buck-boost



Solution

$$\frac{V^2}{R} = P$$

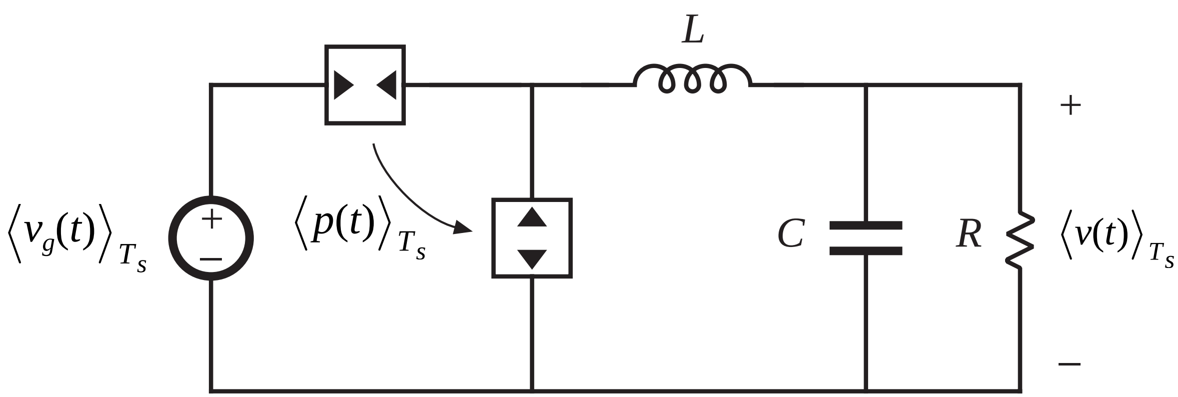
$$P = \frac{\frac{1}{2} L I_c^2(t) f_s}{\left(1 + \frac{M_a}{M_1}\right)^2}$$

$$V = \sqrt{PR} = I_c \sqrt{\frac{RL f_s}{2\left(1 + \frac{M_a}{M_1}\right)^2}}$$

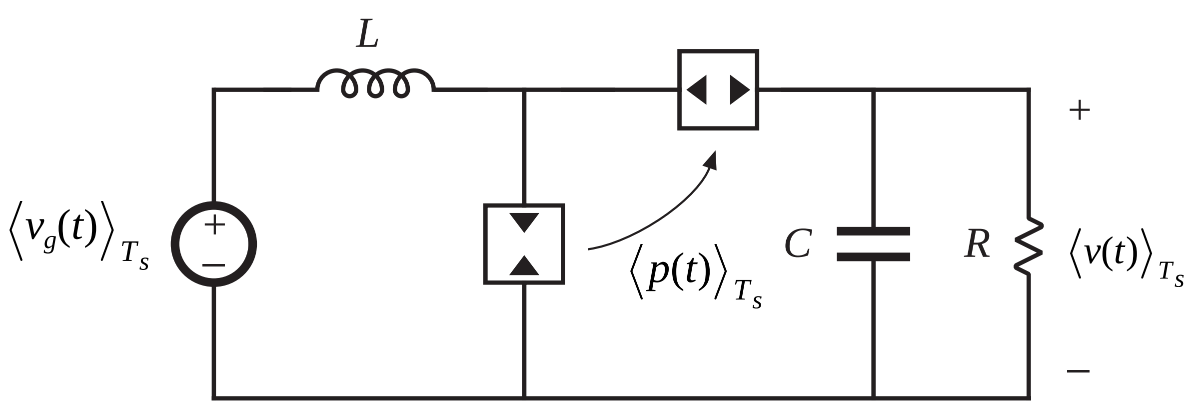
for a resistive load

Models of buck and boost

Buck



Boost



Summary of steady-state DCM CPM characteristics

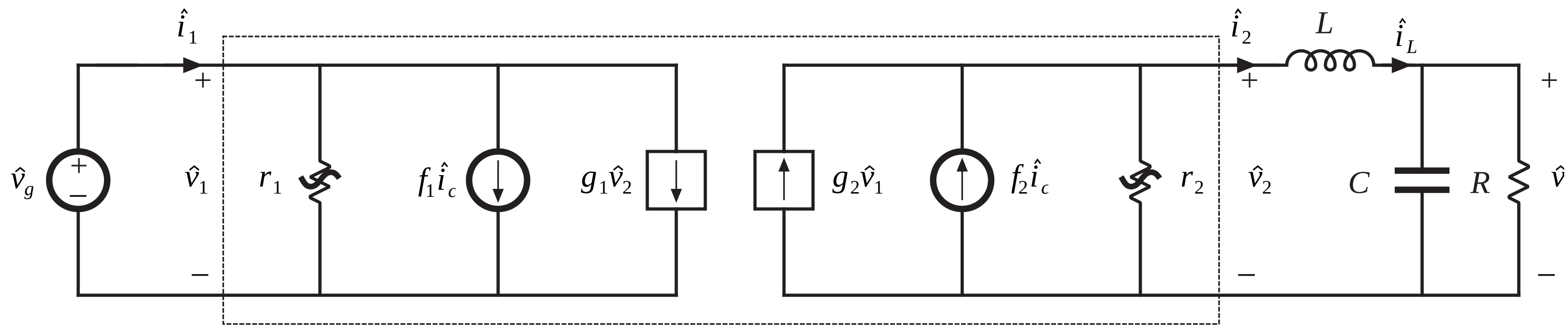
Converter	M	I_{crit}	Stability range when $m_a = 0$
Buck	$\dfrac{P_{load} - P}{P_{load}}$	$\frac{1}{2}\left(I_c - M\, m_a\, T_s\right)$	$0 \le M < \frac{2}{3}$
Boost	$\dfrac{P_{load}}{P_{load} - P}$	$\dfrac{\left(I_c - \frac{M-1}{M}\, m_a\, T_s\right)}{2\,M}$	$0 \le D \le 1$
Buck-boost	Depends on load characteristic: $P_{load} = P$	$\dfrac{\left(I_c - \frac{M}{M-1}\, m_a\, T_s\right)}{2\left(M-1\right)}$	$0 \le D \le 1$

$\left| I \right| > \left| I_{crit} \right|$ for CCM

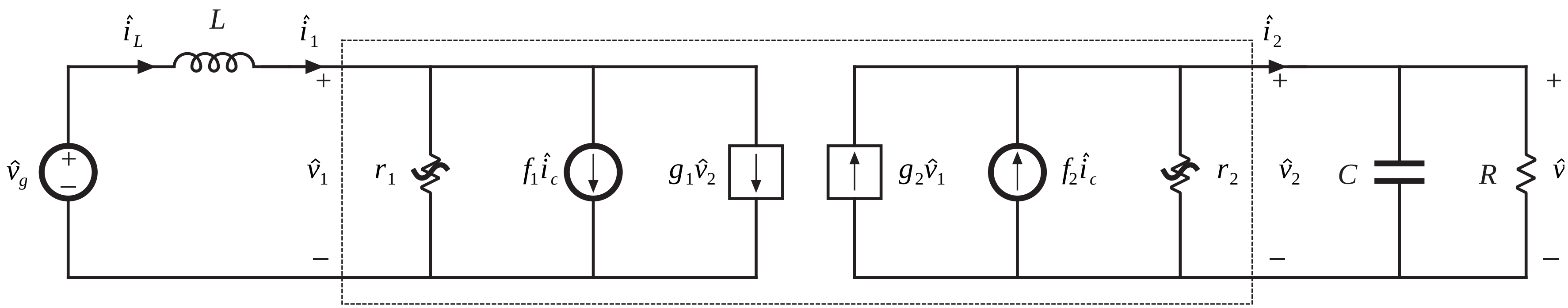
$\left| I \right| < \left| I_{crit} \right|$ for DCM

Linearized small-signal models

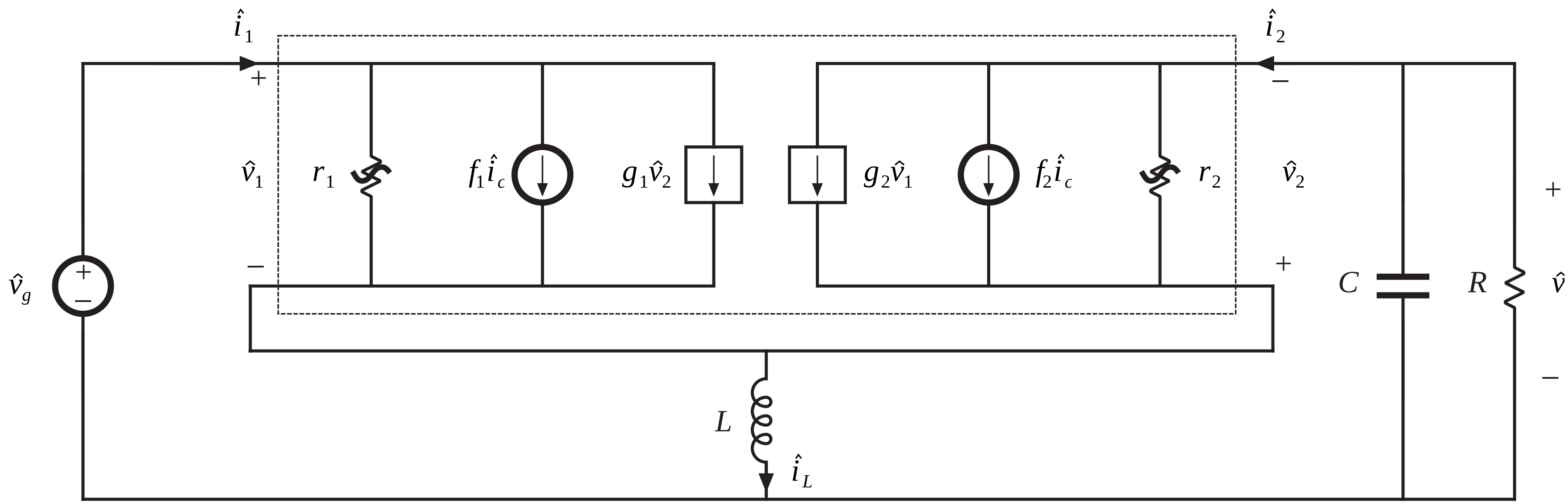
Buck



Boost



Linearized small-signal models: Buck-boost



DCM CPM small-signal parameters: input port

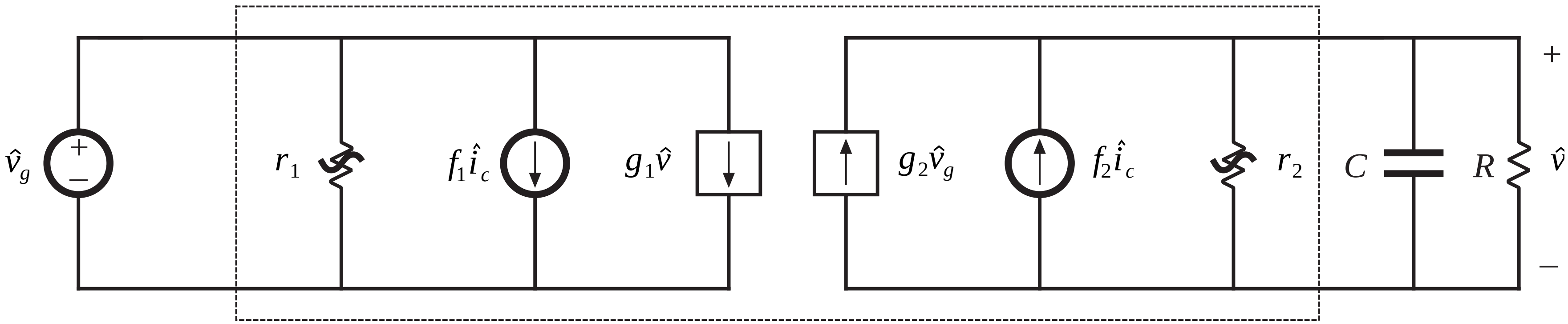
Converter	g_1	f_1	r_1
Buck	$\frac{1}{R}\left(\frac{M^2}{1-M}\right)\frac{\left(1-\frac{m_a}{m_1}\right)}{\left(1+\frac{m_a}{m_1}\right)}$	$2\,\frac{I_1}{I_c}$	$-R\left(\frac{1-M}{M^2}\right)\frac{\left(1+\frac{m_a}{m_1}\right)}{\left(1-\frac{m_a}{m_1}\right)}$
Boost	$-\frac{1}{R}\left(\frac{M}{M-1}\right)$	$2\,\frac{I}{I_c}$	$\frac{R}{M^2\left(\frac{2-M}{M-1}+\frac{2\,\frac{m_a}{m_1}}{1+\frac{m_a}{m_1}}\right)}$
Buck-boost	0	$2\,\frac{I_1}{I_c}$	$\frac{-R}{M^2}\,\frac{\left(1+\frac{m_a}{m_1}\right)}{\left(1-\frac{m_a}{m_1}\right)}$

DCM CPM small-signal parameters: output port

Converter	g_2	f_2	r_2
Buck	$\frac{1}{R}\left(\frac{M}{1-M}\right)\frac{\left(\frac{m_a}{m_1}(2-M)-M\right)}{\left(1+\frac{m_a}{m_1}\right)}$	$2\,\frac{I}{I_c}$	$R\,\frac{(1-M)\left(1+\frac{m_a}{m_1}\right)}{\left(1-2M+\frac{m_a}{m_1}\right)}$
Boost	$\frac{1}{R}\left(\frac{M}{M-1}\right)$	$2\,\frac{I_2}{I_c}$	$R\left(\frac{M-1}{M}\right)$
Buck-boost	$\frac{2M}{R}\,\frac{\left(\frac{m_a}{m_1}\right)}{\left(1+\frac{m_a}{m_1}\right)}$	$2\,\frac{I_2}{I_c}$	R

Simplified DCM CPM model, with $L = 0$

Buck, boost, buck-boost all become



$$G_{vc}(s) = \left.\frac{\hat{v}}{\hat{i}_c}\right|_{\hat{v}_g = 0} = \frac{G_{c0}}{1 + \frac{s}{\omega_p}}$$

$$G_{vg}(s) = \left.\frac{\hat{v}}{\hat{v}_g}\right|_{\hat{i}_c = 0} = \frac{G_{g0}}{1 + \frac{s}{\omega_p}}$$

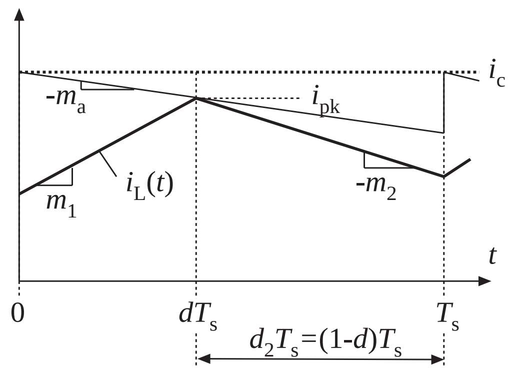
$$G_{c0} = f_2 \left(R \,||\, r_2\right)$$

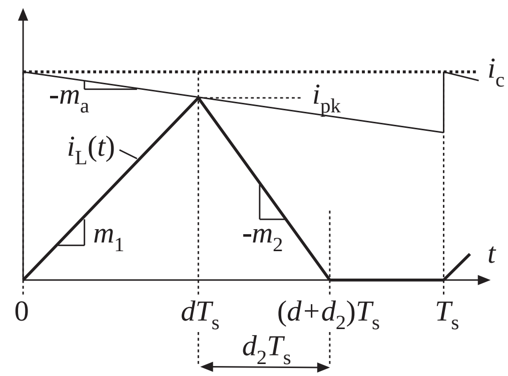
$$\omega_p = \frac{1}{\left(R \,||\, r_2\right) C}$$

$$G_{g0} = g_2 \left(R \,||\, r_2\right)$$

Current-Programmed (CPM) Controller: Large-Signal CCM/DCM Averaged Model

$$i_{pk} = i_c - m_a dT_s$$





$$\langle i_L \rangle_{T_s} = d\left(i_{pk} - \frac{m_1 dT_s}{2}\right) + d_2\left(i_{pk} - \frac{m_2 d_2 T_s}{2}\right)$$

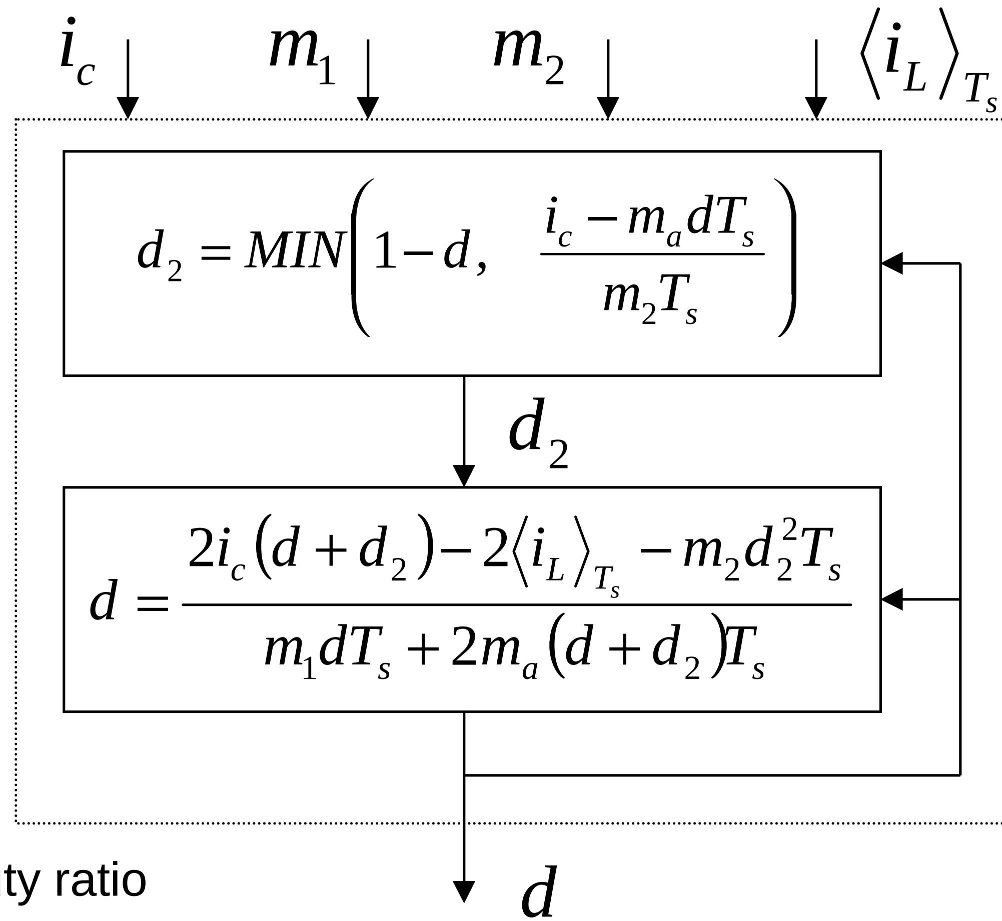
$$d_2 = \begin{cases} 1-d & CCM \\ \dfrac{i_{pk}}{m_2 T_s} & DCM \end{cases}$$

CCM/DCM: $d_2 = MIN\left(1-d, \quad \dfrac{i_{pk}}{m_2 T_s}\right)$

CPM Controller: Large-Signal CCM/DCM Averaged Model

Inputs:

Model:



Output: duty ratio

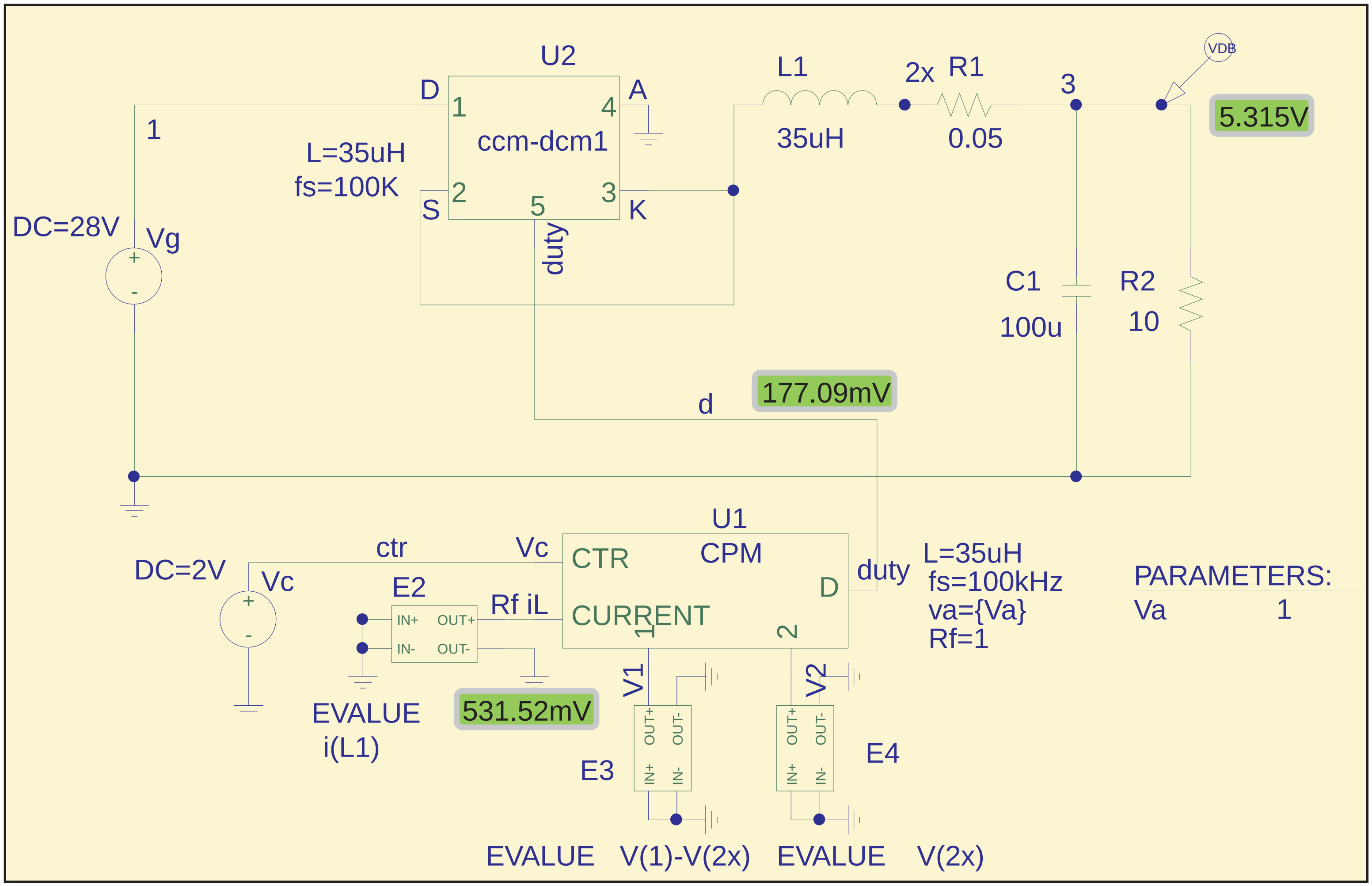
CPM Large-Signal Averaged Model: PSpice Implementation

```
************************************************************
* MODEL: CPM
* Current-Programmed-Mode CCM/DCM controller model.
* All parameters and inputs are referred to
* the primary side.
************************************************************
* Parameters:
*     L=equivalent inductance, referred to primary
*     fs=switching frequency
*     va=amplitude of the artificial ramp, va=Rf*ma/fs
*     Rf=equivalent current-sense resistance
************************************************************
* Nodes:
* ctr: control input, v(ctr)=Rf*ic
* current: sensed average inductor current v(current)=Rf*iL
* 1: voltage across L in interval 1, slope m1=v(1)/L
* 2: (-) voltage across L in interval 2, slope m2=v(2)/L
* d: duty ratio (output of the controller)
************************************************************
.subckt CPM ctr current 1 2 d
+params: L=100e-6 fs=1e5 va=0.5 Rf=0.1
*
* generate d2 for CCM/DCM
Ed2 d2 0 table
+ {MIN(
+ L*fs*(v(ctr)-va*v(d))/Rf/(v(2)),
+ 1-v(d)
+ )} (0,0) (1,1)
*
Em1 m1 0 value={Rf*v(1)/L/fs}
Em2 m2 0 value={Rf*v(2)/L/fs}
*
* generate duty-ratio d (valid CCM and DCM operation)
*
Eduty d 0 table
+ {
+ 2*(v(ctr)*(v(d)+v(d2))
+ -v(current)-v(m2)*v(d2)*v(d2)/2)
+ /(v(m1)*v(d)+2*va*(v(d)+v(d2)))
+ } (0.01,0.01) (0.99,0.99)
*
.ends ; end of subcircuit CPM
************************************************************
```

Application Example:
Buck Converter with Current-Mode Control

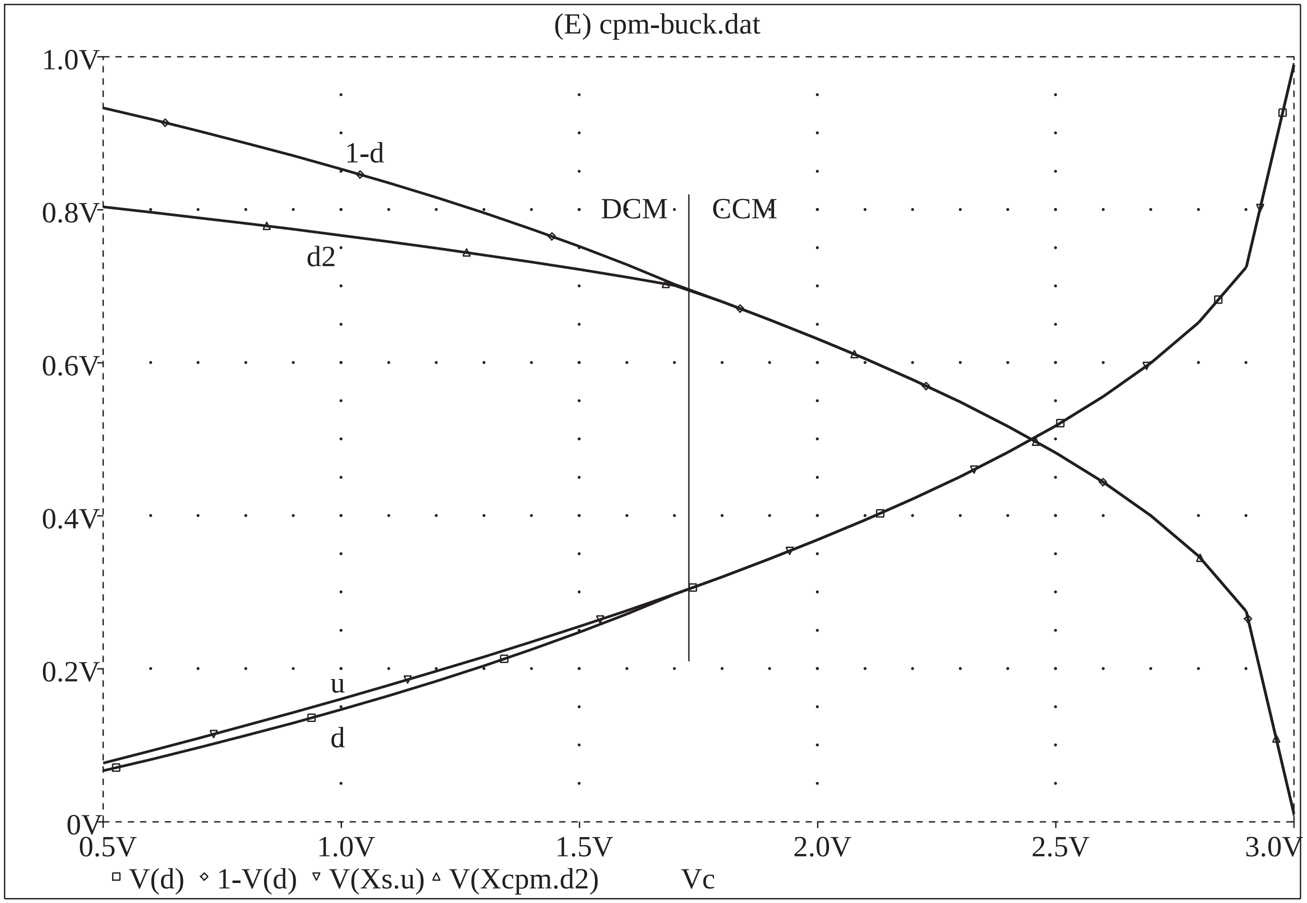
- Demonstrate how CCM/DCM averaged-switch model can be used together with CCM/DCM averaged model of the current-mode controller
- Use DC sweep simulation to show steady-state characteristics including operation in DCM or CCM
- Use AC simulation to show control-to-output responses compared for duty-ratio control and current-mode control, in DCM or CCM
- Use parametric sweep simulation to find the amplitude of the artificial ramp to minimize input-to-output audio-susceptibility
- Specifications:
  - Input V_g = 28V, output V = 5-20V, 0.5-2A
  - Switching frequency f_s=100kHz, inductance L = 35uH
  - Equivalent current-sense resistance R_f = 1
  - Artificial-ramp amplitude V_a = 0-3V

Buck Converter with Current-Mode Control



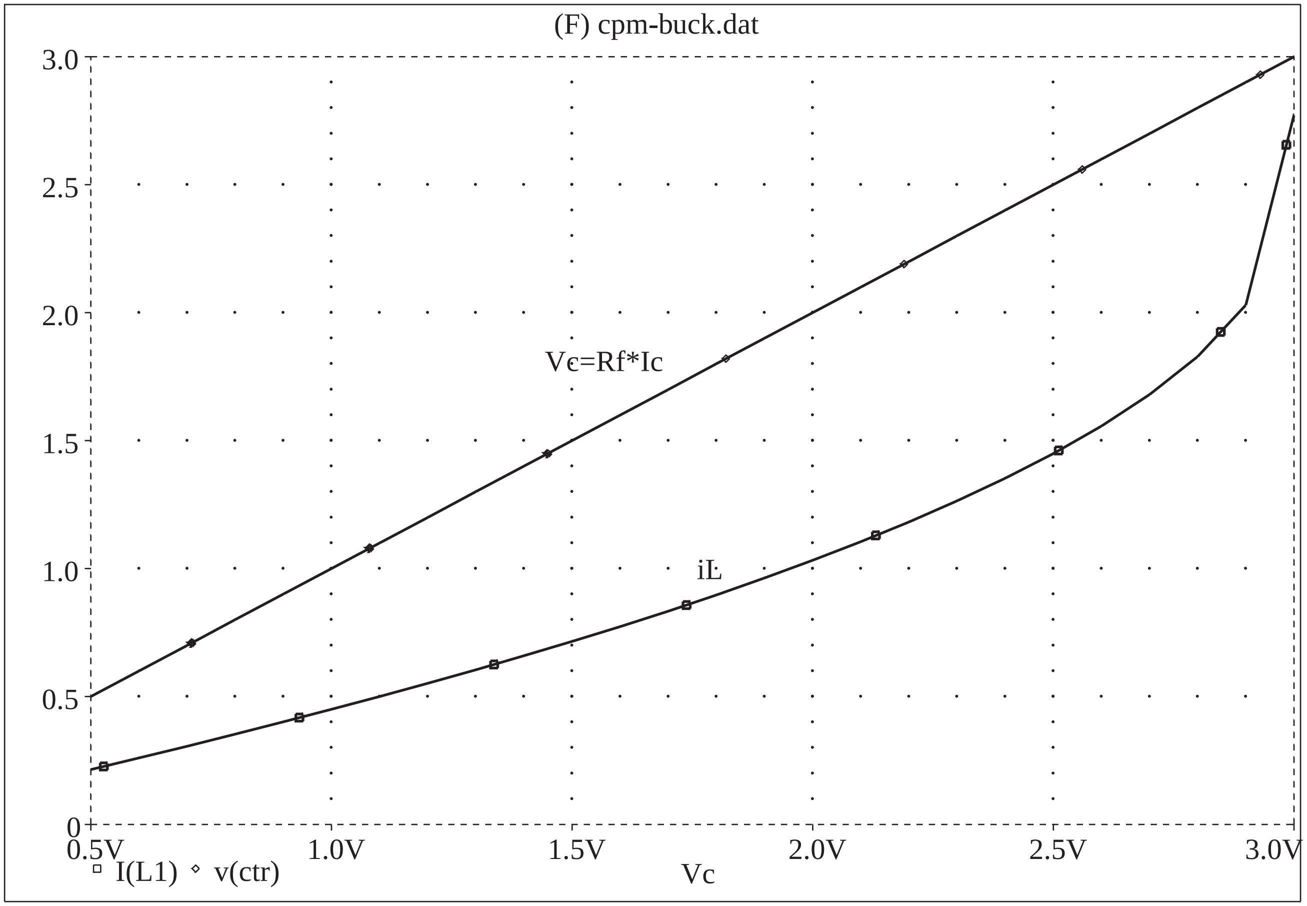
Example: Cpm-buck

Buck Converter with Current-Mode Control DC Sweep Simulation



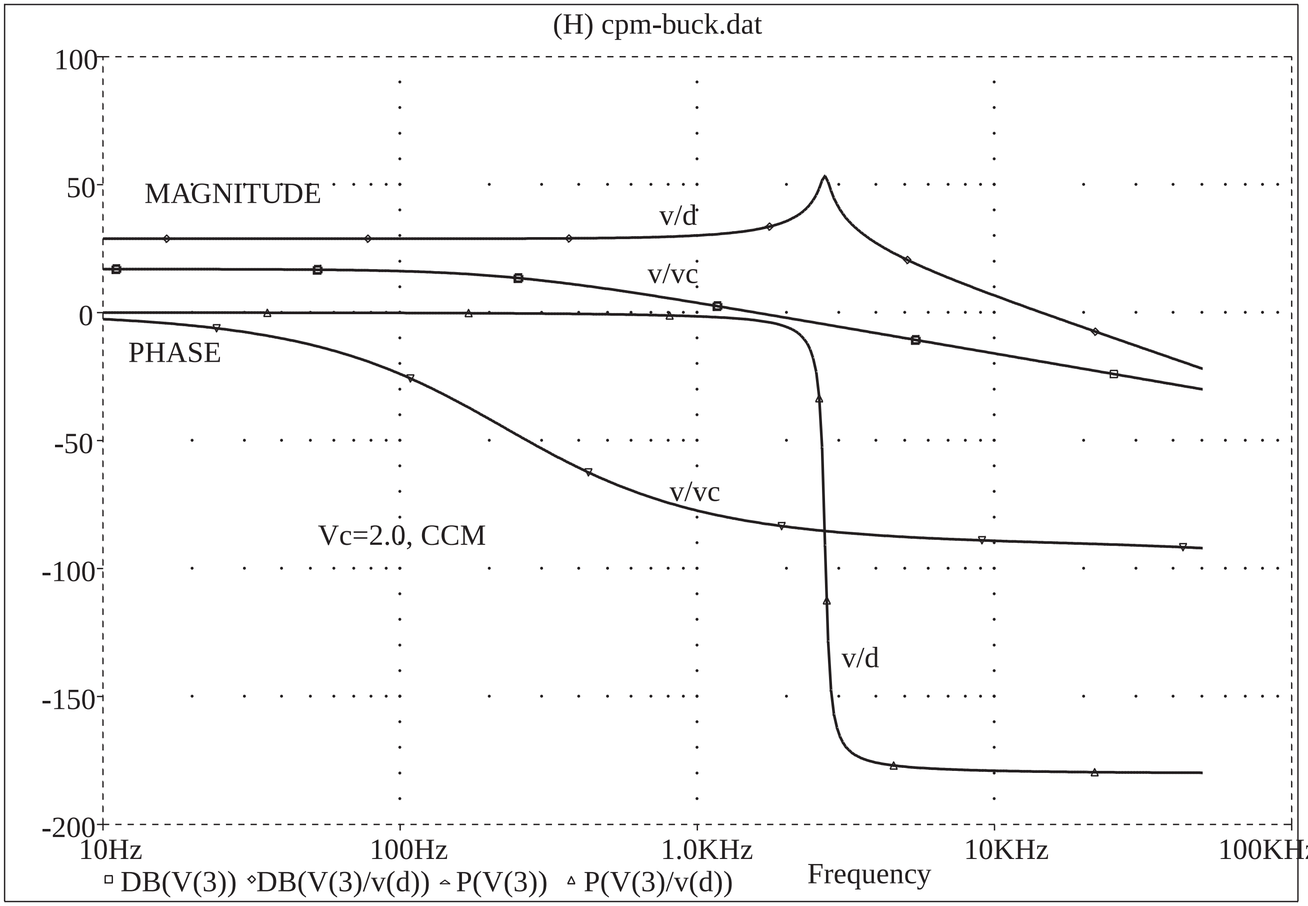
Duty ratio d, equivalent duty ratio u, and diode conduction interval d_2 as functions of the control input V_c

Buck Converter with Current-Mode Control
DC Sweep Simulation



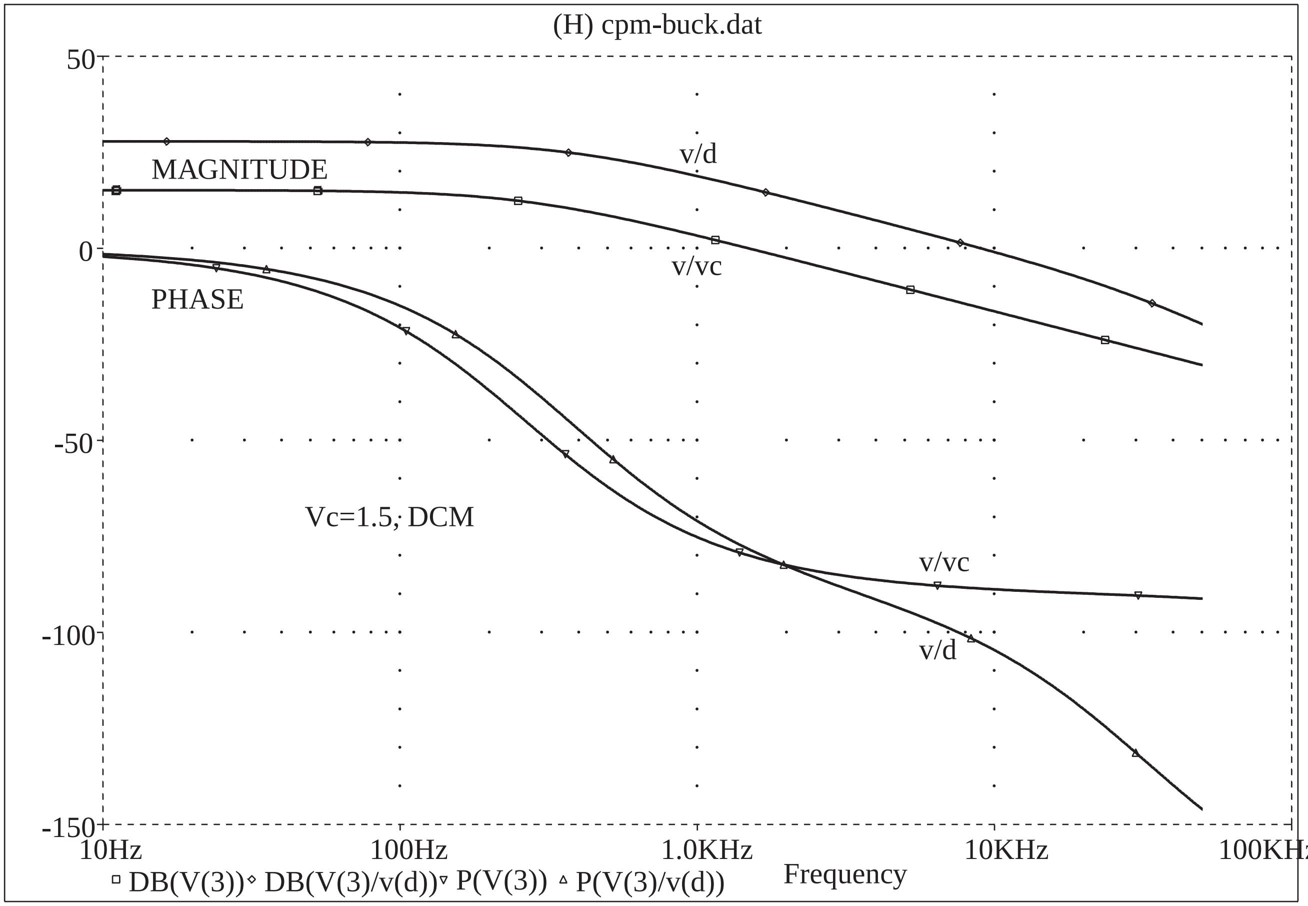
Average inductor current iL as a function of the control input V_c

Buck Converter with Current-Mode Control
Frequency Responses in CCM



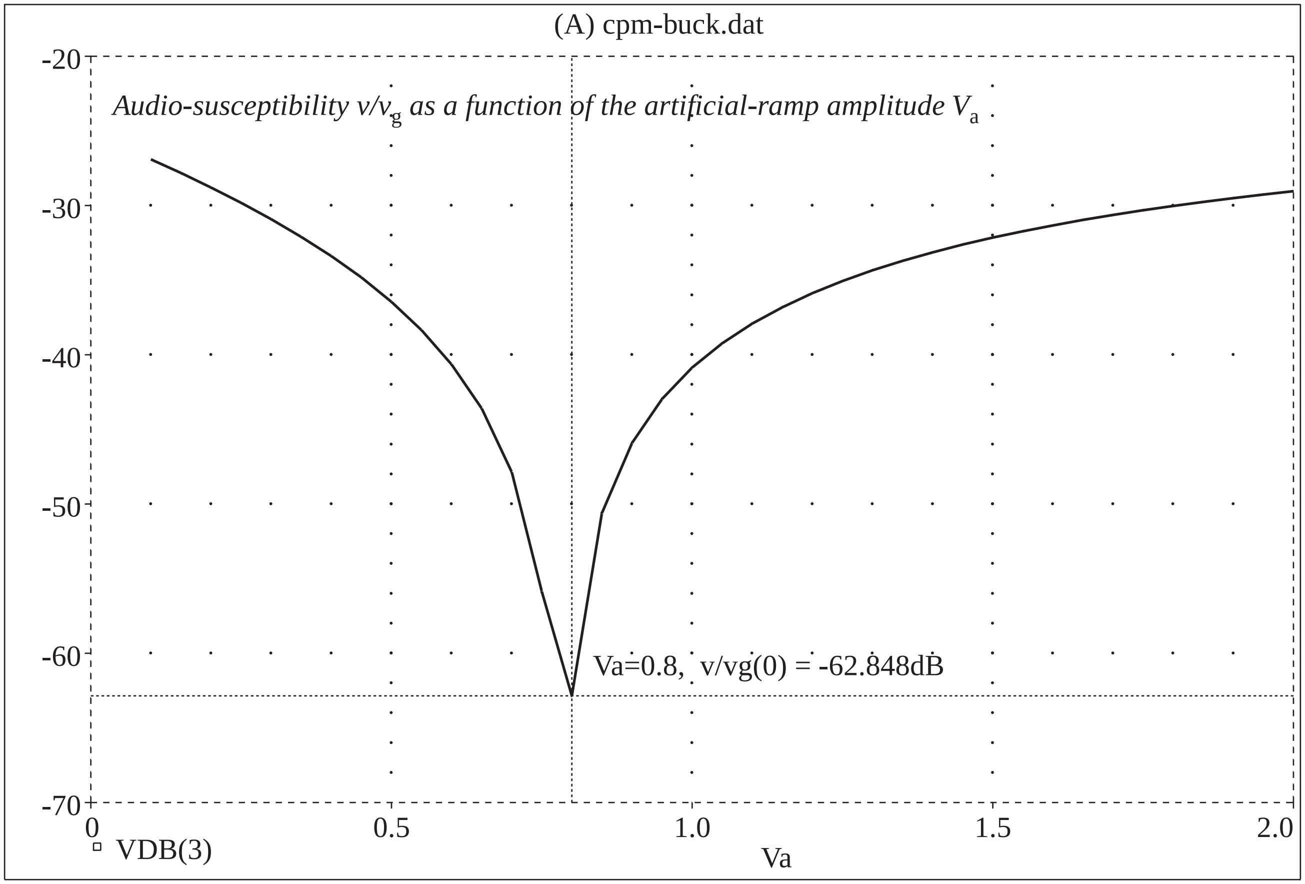
Control-to-output frequency responses for duty-ratio control (*v/d*) and current-mode control (v/v_c). The converter operates in CCM.

Buck Converter with Current-Mode Control
Frequency Responses in DCM



Control-to-output frequency responses for duty-ratio control (*v/d*) and current-mode control (v/v_c). The converter operates in DCM.

Buck Converter with Current-Mode Control
Audio-susceptibility Analysis



Parametric sweep used to determine amplitude of the artificial ramp V_a to minimize input-to-output response (audio-susceptibility) v/v_g.

5. Averaged modeling of single-phase low-harmonic rectifiers

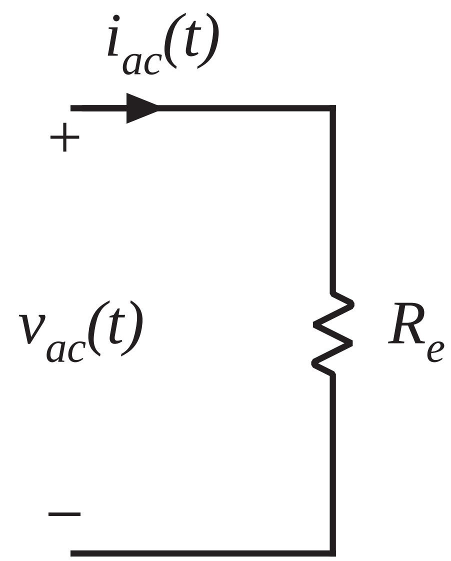
- **Ideal rectifier**
- **Averaged model obtained by averaging over switching period**
- **Averaged model obtained by averaging over line period**
- **Application examples:**
  - Power factor corrector based on boost converter operating in DCM
  - Power factor corrector based on SEPIC with nonlinear-carrier control

Properties of the Ideal Rectifier

It is desired that the rectifier present a resistive load to the ac power system. This leads to

- unity power factor
- ac line current has same waveshape as voltage

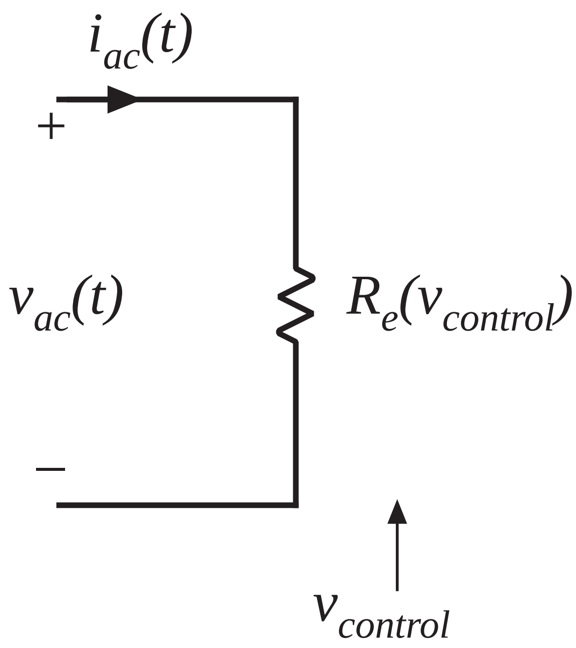
$$i_{ac}(t) = \frac{v_{ac}(t)}{R_e}$$



R_e is called the *emulated resistance*

Control of power throughput

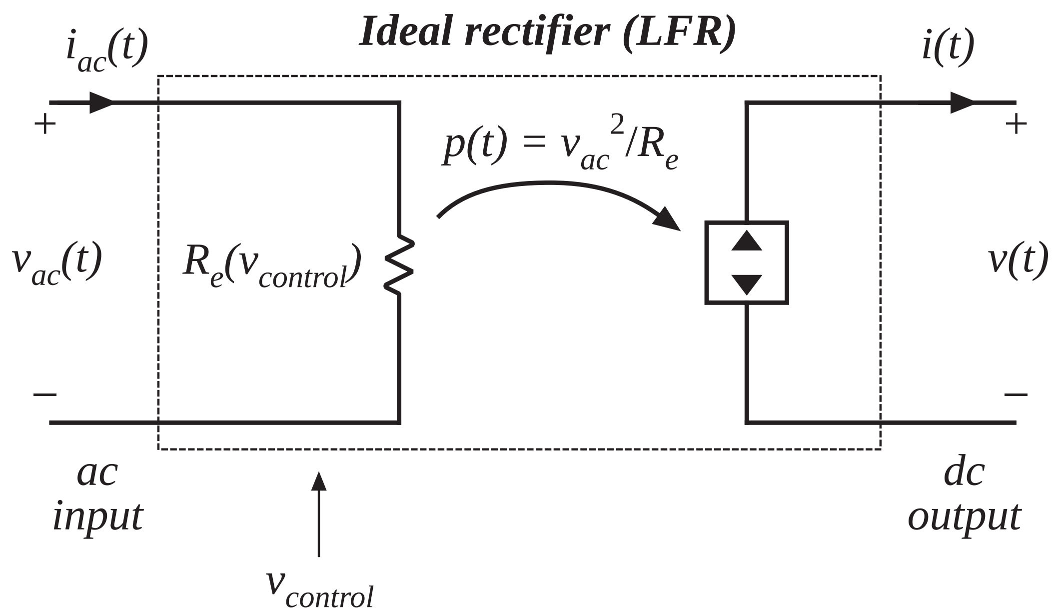
$$P_{av} = \frac{V_{ac,rms}^2}{R_e(v_{control})}$$



Power apparently "consumed" by R_e is actually transferred to rectifier dc output port. To control the amount of output power, it must be possible to adjust the value of R_e.

Output port model

The ideal rectifier is lossless and contains no internal energy storage. Hence, the instantaneous input power equals the instantaneous output power. Since the instantaneous power is independent of the dc load characteristics, the output port obeys a power source characteristic.



$$p(t) = \frac{v_{ac}^2(t)}{R_e(v_{control}(t))} \qquad v(t)i(t) = p(t) = \frac{v_{ac}^2(t)}{R_e}$$

Equations of the ideal rectifier / LFR

Defining equations of the ideal rectifier:

$$i_{ac}(t) = \frac{v_{ac}(t)}{R_e(v_{control})}$$

$$v(t)i(t) = p(t)$$

$$p(t) = \frac{v_{ac}^2(t)}{R_e(v_{control}(t))}$$

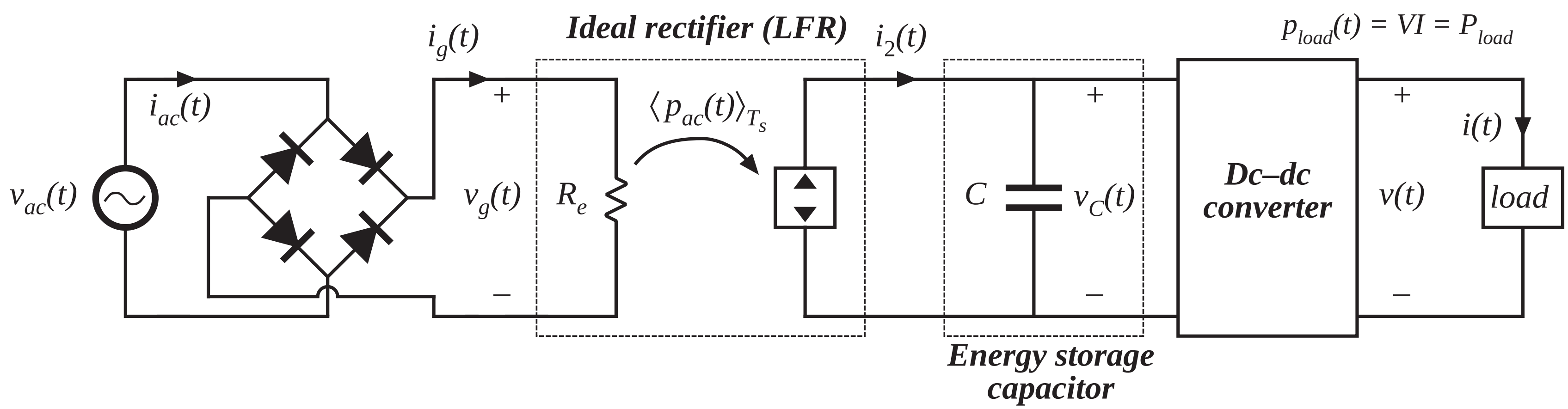
When connected to a resistive load of value R, the input and output rms voltages and currents are related as follows:

$$\frac{V_{rms}}{V_{ac,rms}} = \sqrt{\frac{R}{R_e}}$$

$$\frac{I_{ac,rms}}{I_{rms}} = \sqrt{\frac{R}{R_e}}$$

A switch network that is capable of satisfying the above (averaged) equations can be employed in low-harmonic rectifier applications

Single-phase system with internal energy storage



Energy storage capacitor voltage $v_C(t)$ must be independent of input and output voltage waveforms, so that it can vary according to

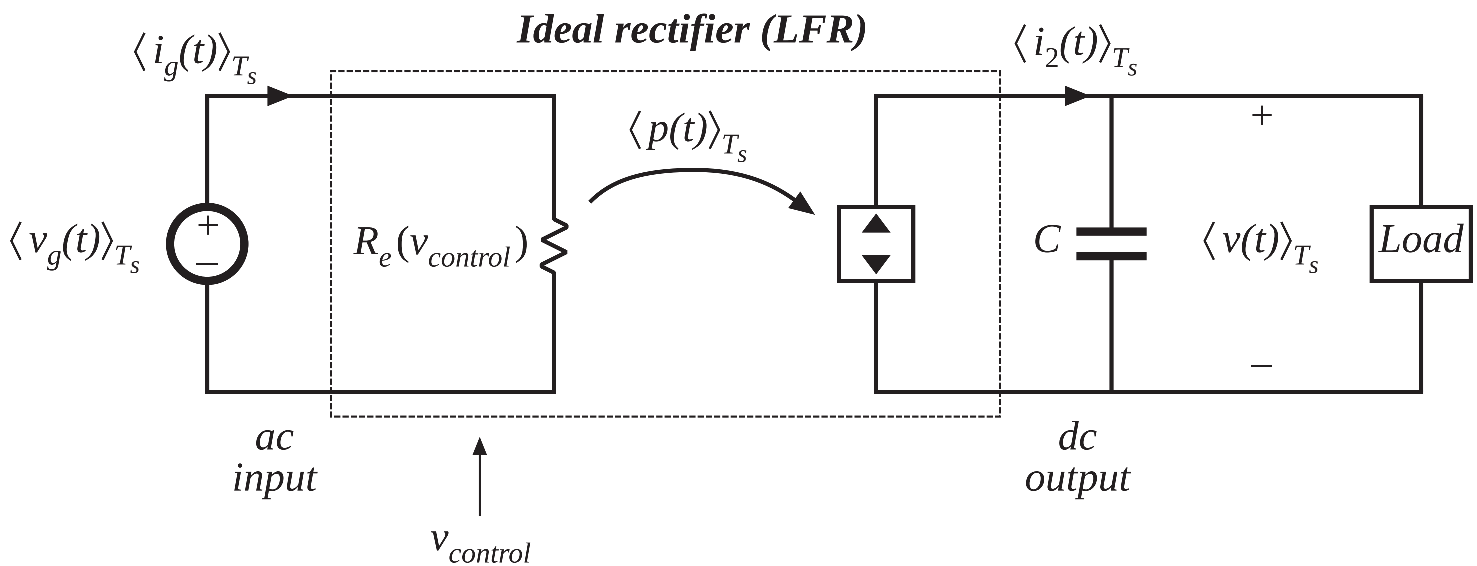
$$\frac{d\left(\frac{1}{2}Cv_C^2(t)\right)}{dt} = p_{ac}(t) - p_{load}(t)$$

This system is capable of

- Wide-bandwidth control of output voltage
- Wide-bandwidth control of input current waveform
- Internal independent energy storage

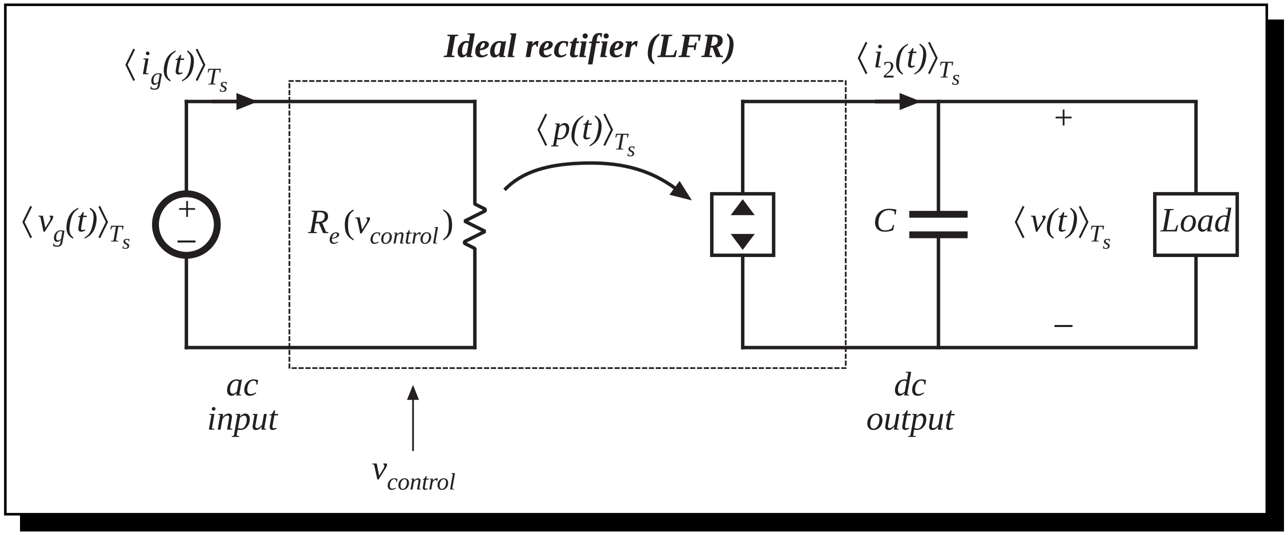
Large signal model

averaged over switching period T_s



- Ideal rectifier model, assuming that inner wide-bandwidth loop operates ideally
- High-frequency switching harmonics are removed via averaging
- Ac line-frequency harmonics are included in model
- Nonlinear and time-varying

Predictions of large-signal model



If the input voltage is

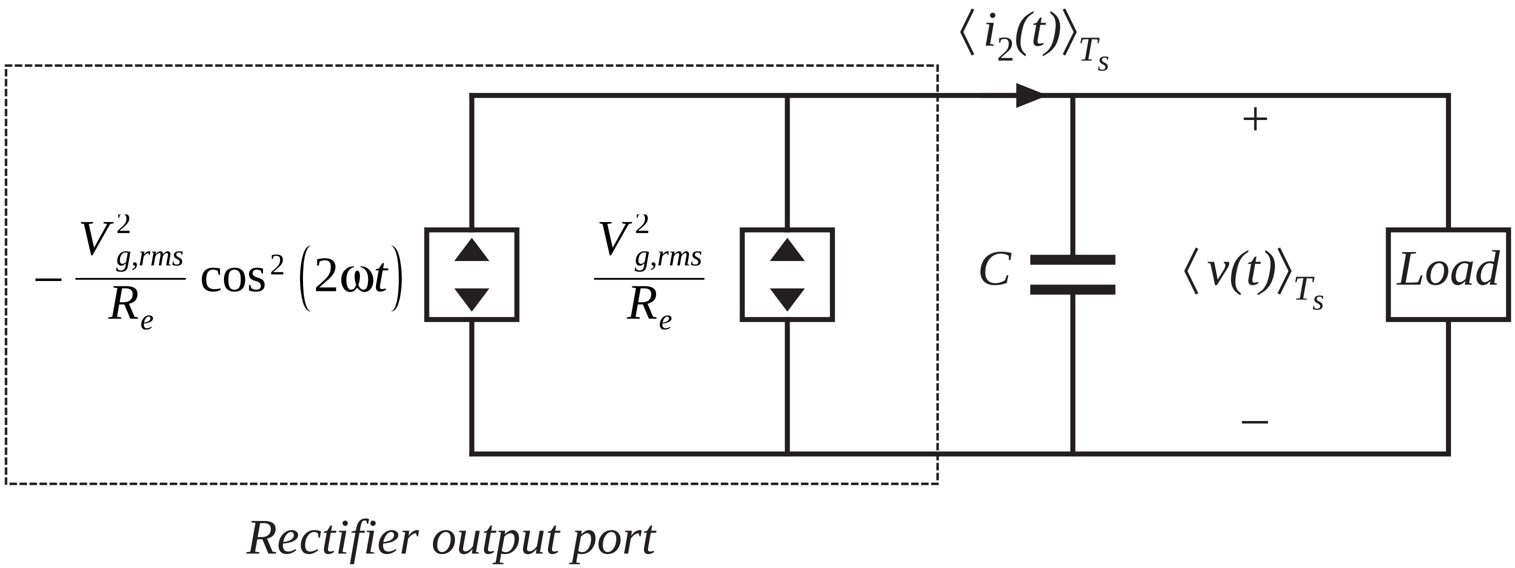
$$v_g(t) = \sqrt{2}\, v_{g,rms} \left| \sin\left(\omega t\right) \right|$$

Then the instantaneous power is:

$$\left\langle p(t) \right\rangle_{T_s} = \frac{\left\langle v_g(t) \right\rangle_{T_s}^2}{R_e(v_{control}(t))} = \frac{v_{g,rms}^2}{R_e(v_{control}(t))} \left(1 - \cos\left(2\omega t\right) \right)$$

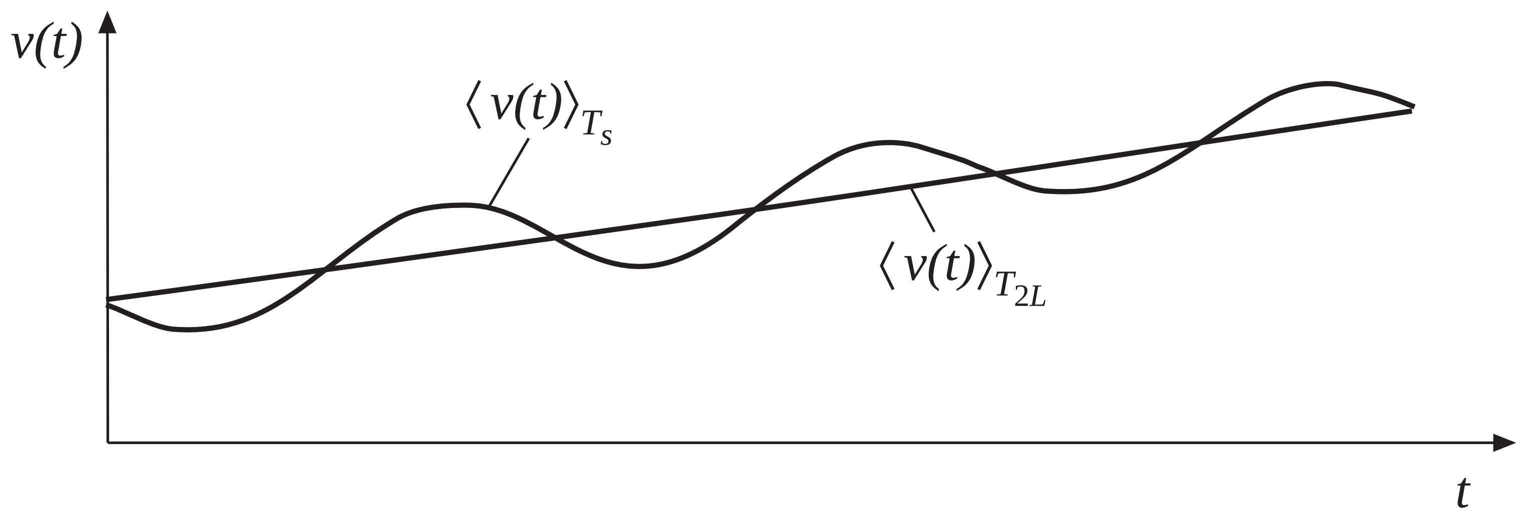
which contains a constant term plus a second-harmonic term

Separation of power source into its constant and time-varying components



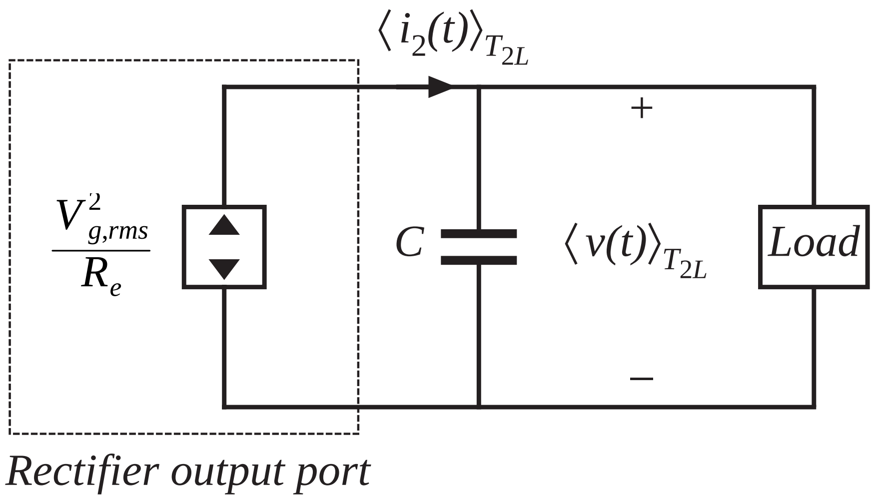
The second-harmonic variation in power leads to second-harmonic variations in the output voltage and current

Removal of even harmonics via averaging



$$T_{2L} = \frac{1}{2}\,\frac{2\pi}{\omega} = \frac{\pi}{\omega}$$

Resulting averaged model



Time invariant model

Power source is nonlinear

Perturbation and linearization

The averaged model predicts that the rectifier output current is

$$\left\langle i_2(t) \right\rangle_{T_{2L}} = \frac{\left\langle p(t) \right\rangle_{T_{2L}}}{\left\langle v(t) \right\rangle_{T_{2L}}} = \frac{v_{g,rms}^2(t)}{R_e(v_{control}(t)) \left\langle v(t) \right\rangle_{T_{2L}}}$$

$$= f\left(v_{g,rms}(t), \left\langle v(t) \right\rangle_{T_{2L}}, v_{control}(t)) \right)$$

Let

$$\left\langle v(t) \right\rangle_{T_{2L}} = V + \hat{v}(t)$$

$$\left\langle i_2(t) \right\rangle_{T_{2L}} = I_2 + \hat{i}_2(t)$$

$$v_{g,rms} = V_{g,rms} + \hat{v}_{g,rms}(t)$$

$$v_{control}(t) = V_{control} + \hat{v}_{control}(t)$$

with

$$V >> \left| \hat{v}(t) \right|$$

$$I_2 >> \left| \hat{i}_2(t) \right|$$

$$V_{g,rms} >> \left| \hat{v}_{g,rms}(t) \right|$$

$$V_{control} >> \left| \hat{v}_{control}(t) \right|$$

Linearized result

$$I_2 + \hat{i}_2(t) = g_2 \hat{v}_{g,rms}(t) + j_2 \hat{v}(t) - \frac{\hat{v}_{control}(t)}{r_2}$$

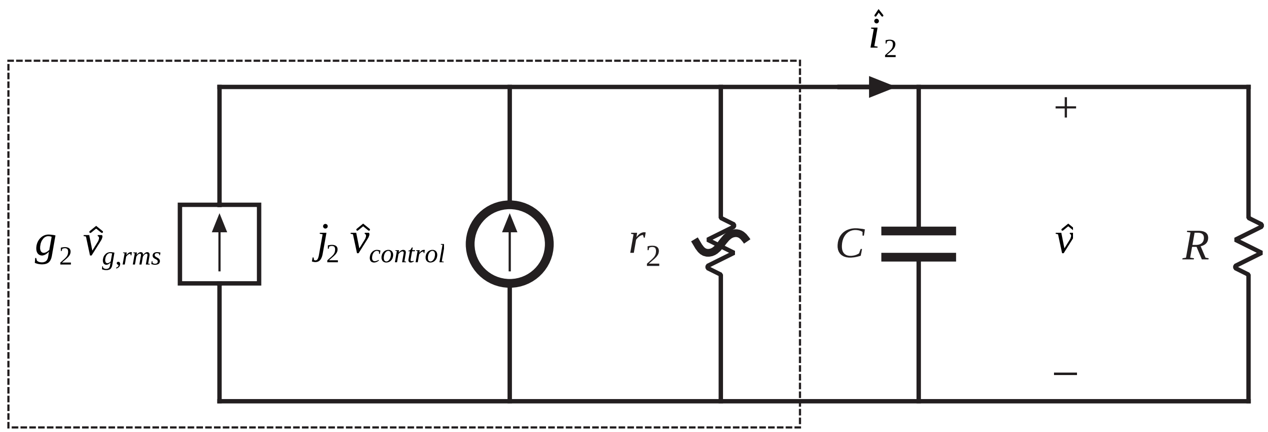
where

$$g_2 = \left. \frac{df\left(v_{g,rms},\, V,\, V_{control})\right)}{dv_{g,rms}} \right|_{v_{g,rms} = V_{g,rms}} = \frac{2}{R_e(V_{control})} \frac{V_{g,rms}}{V}$$

$$\left(-\frac{1}{r_2}\right) = \left. \frac{df\left(V_{g,rms},\, \left\langle v \right\rangle_{T_{2L}},\, V_{control})\right)}{d\left\langle v \right\rangle_{T_{2L}}} \right|_{\left\langle v \right\rangle_{T_{2L}} = V} = -\frac{I_2}{V}$$

$$j_2 = \left. \frac{df\left(V_{g,rms},\, V,\, v_{control})\right)}{dv_{control}} \right|_{v_{control} = V_{control}} = -\frac{V_{g,rms}^2}{V R_e^2(V_{control})} \left. \frac{dR_e(v_{control})}{dv_{control}} \right|_{v_{control} = V_{control}}$$

Small-signal equivalent circuit



Rectifier output port

Predicted transfer functions

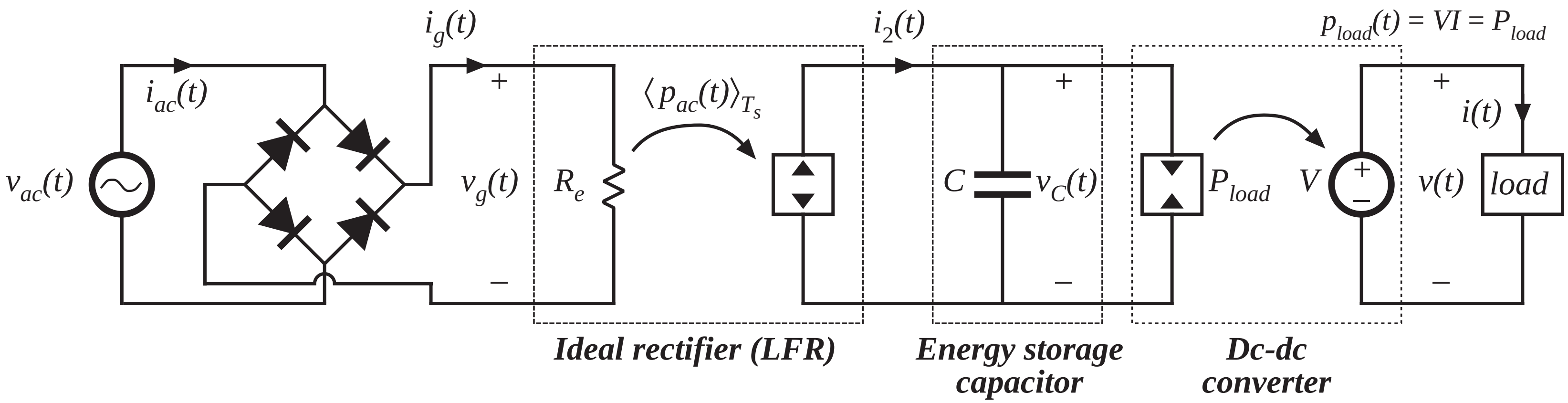
Control-to-output

$$\frac{\hat{v}(s)}{\hat{v}_{control}(s)} = j_2\, R \| r_2 \, \frac{1}{1 + sC\, R \| r_2}$$

Line-to-output

$$\frac{\hat{v}(s)}{\hat{v}_{g,rms}(s)} = g_2\, R \| r_2 \, \frac{1}{1 + sC\, R \| r_2}$$

Constant power load



Rectifier and dc-dc converter operate with same average power

Incremental resistance R of constant power load is negative, and is

$$R = -\frac{V^2}{P_{av}}$$

which is equal in magnitude and opposite in polarity to rectifier incremental output resistance r_2 for all controllers except NLC

Transfer functions with constant power load

When $r_2 = -R$, the parallel combination $r_2 \parallel R$ becomes equal to zero. The small-signal transfer functions then reduce to

$$\frac{\hat{v}(s)}{\hat{v}_{control}(s)} = \frac{j_2}{sC}$$

$$\frac{\hat{v}(s)}{\hat{v}_{g,rms}(s)} = \frac{g_2}{sC}$$

Application Example: Power-Factor Corrector Based on Boost Converter Operating in DCM

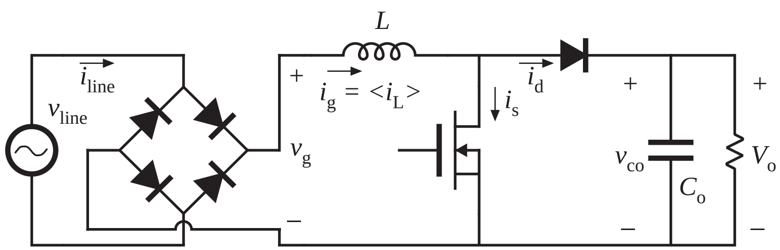
Objectives:

- Example of how large-signal averaged-switch model can be used for analysis and simulation of a power-factor corrector
- Show examples of averaged pulse-width modulator model, and implementation of closed-loop control
- Use transient simulation to study start-up transient response of the PFC and harmonic distortion of the AC line current in steady state

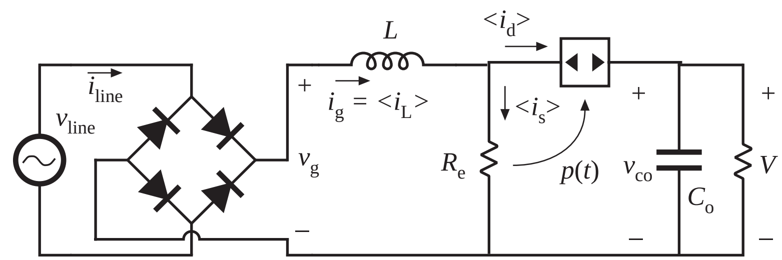
Specifications:

- Input: 120Vrms, 50Hz. Output: 300VDC, 100W
- Switching frequency: 100kHz

DCM Boost PFC



Switching circuit model



Averaged circuit model (in DCM)

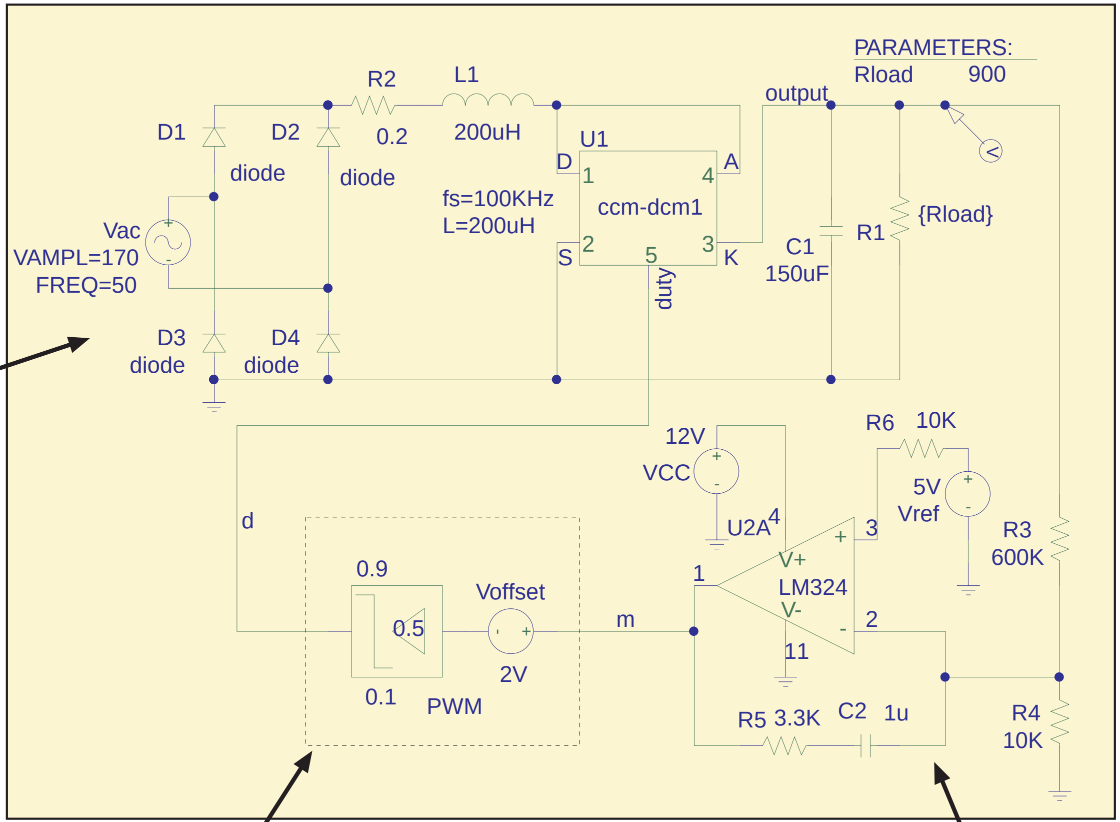
Boost converter operates in DCM at constant duty ratio, constant frequency

$$i_g = \langle i_s \rangle_{T_s} + \langle i_d \rangle_{T_s} = \frac{v_g}{R_e} + \frac{p}{V - v_g} = \frac{v_g}{R_e} + \frac{v_g^2}{R_e(V - v_g)}$$

$$i_g = \frac{v_g}{R_e}\left(\frac{1}{1 - \frac{v_g}{V}}\right) \qquad R_e = \frac{2L}{D^2 T_s}$$

Line current distrortion due to this term

DCM Boost PFC

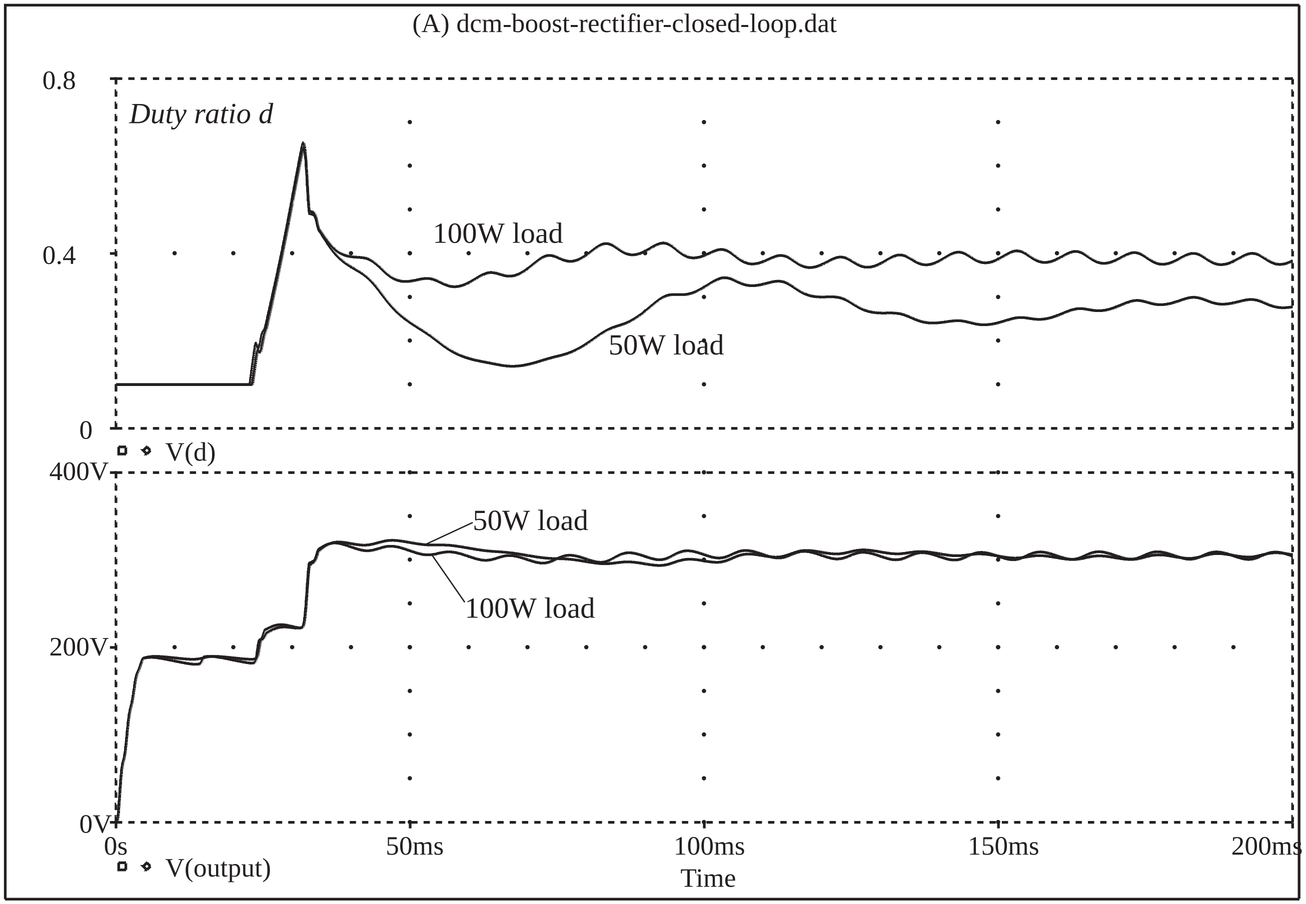


Averaged model of the boost rectifier

Averaged PWM model: $d=v_m/V_M=0.5v_m$, $D_{min}=0.1$, $D_{max}=0.9$ limits

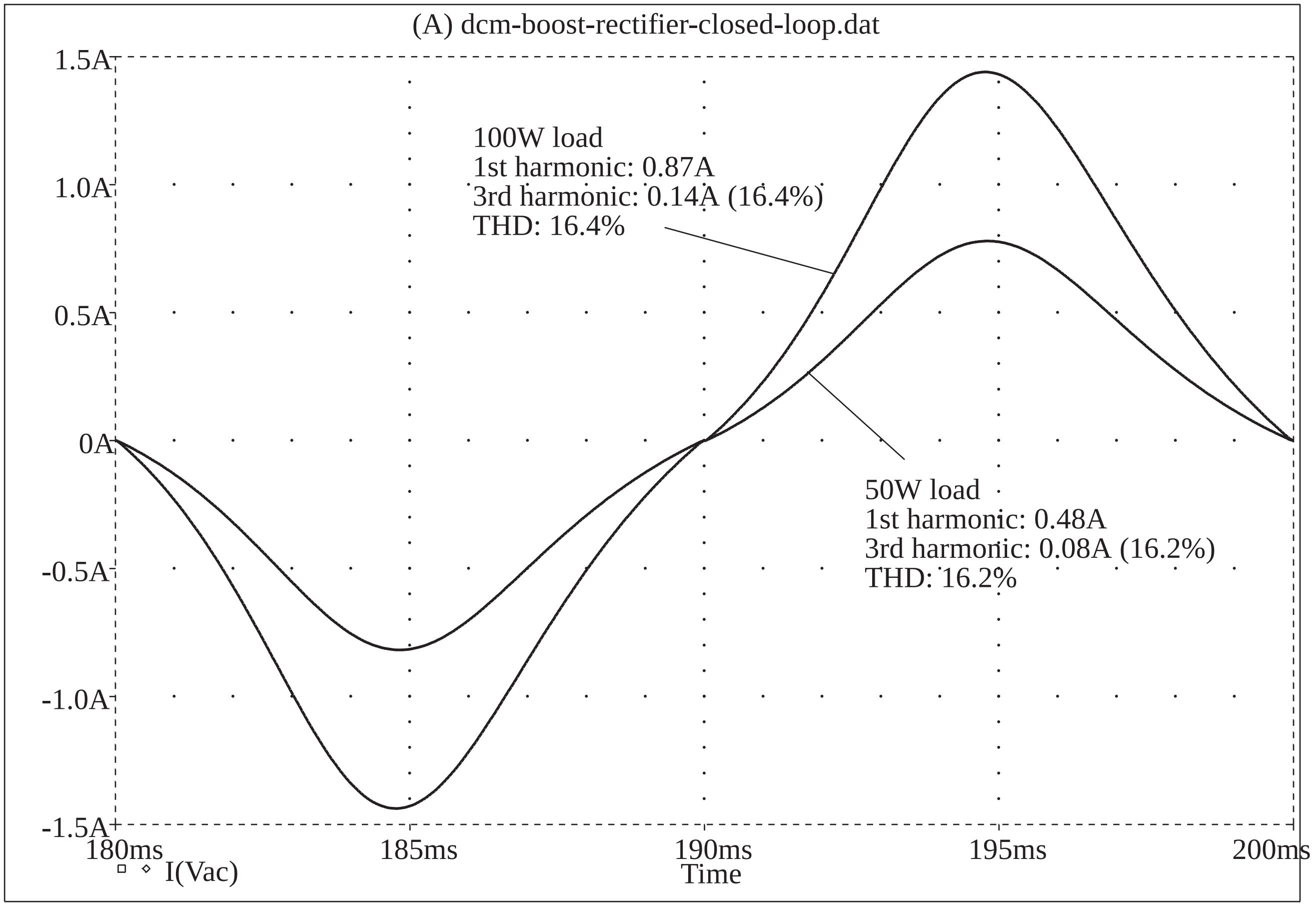
Closed-loop output voltage control

DCM Boost PFC: Start-Up Transient



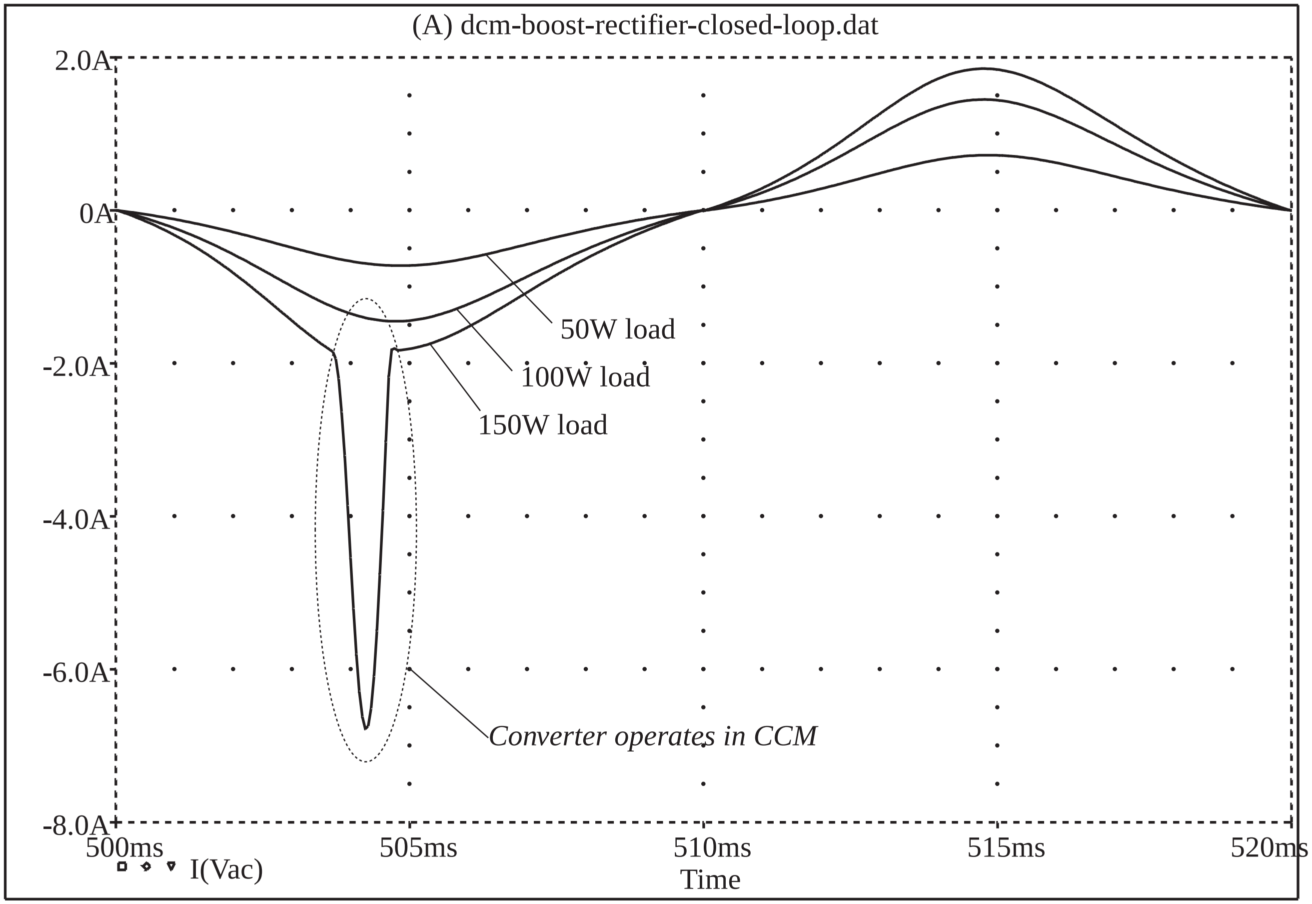
Start-up transient response for full load and 50% load

DCM Boost PFC:
AC Line Current Waveforms



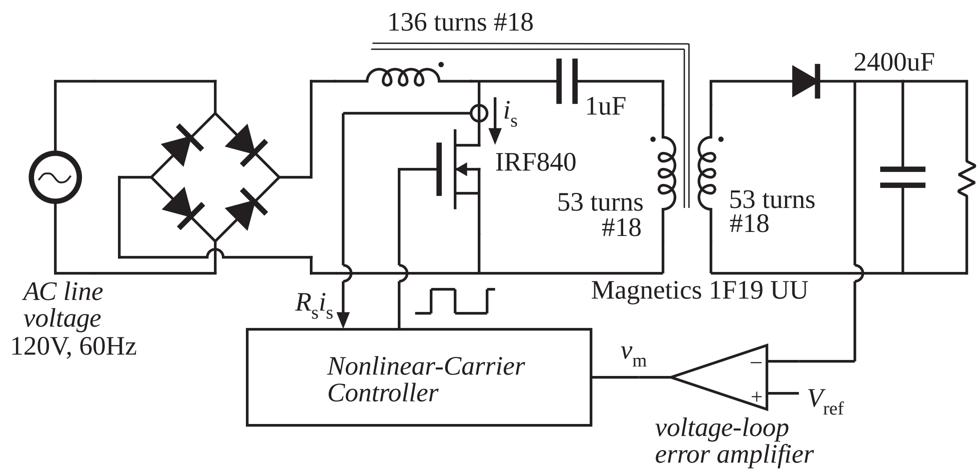
AC line current waveforms at full load and 50% load

DCM Boost PFC:
AC Line Current Distortion in CCM



AC line current waveforms at full load (100W), 50% load, and 150% load

Application Example:
Sepic PFC with NLC Control



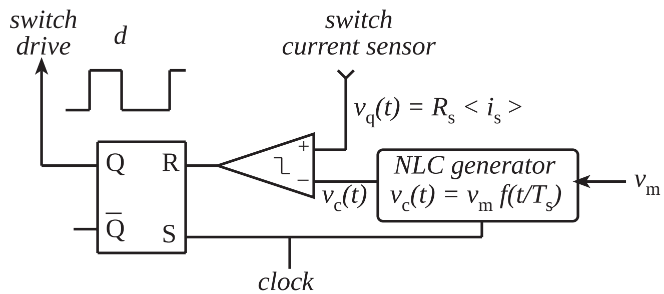
- Active current shaping using Nonlinear Carrier Control method
- Sepic converter has integrated magnetics designed for zero switching ripple in the AC line current
- Specifications:
    - Input: 90-120Vrms, 60Hz. Output: 48VDC, 200W
    - Switching frequency: 90kHz

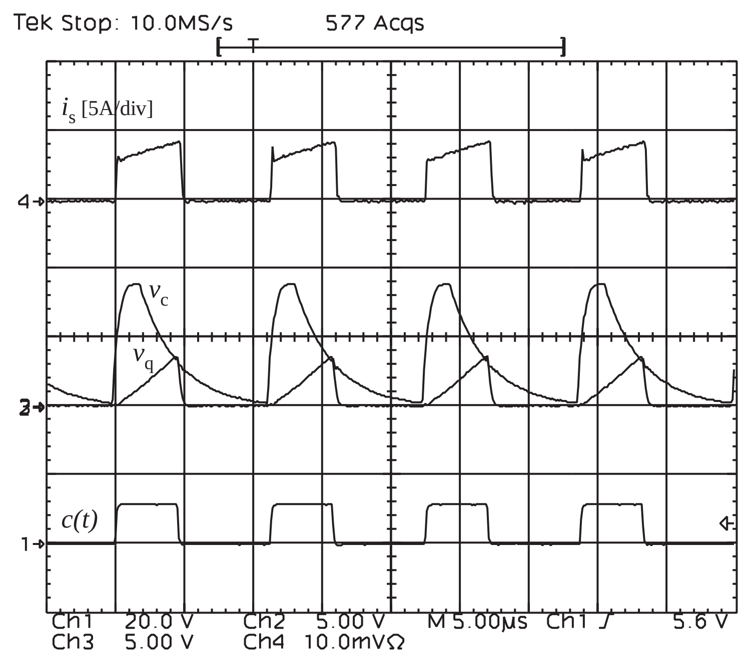
Application Example: Sepic PFC with NLC Control

Objectives:

- Show application of the CCM/DCM averaged-switch model in power-factor correctors with active current shaping and closed-loop output voltage control
- Show average model implementation of a nonlinear pulse-width modulator (NLC controller)
- Compare average model predictions to experimental results:
  - AC line current waveshapes
  - Start-up and load transient responses

NLC Controller Operation





$$f(t/T_s) = v_m \frac{T_s}{t}\left(1 - \frac{t}{T_s}\right) \rightarrow R_s \left\langle i_s \right\rangle_{T_s} = v_m \frac{1-d}{d}$$

$$\frac{1-d}{d} = \frac{v_g}{V}$$

$$i_g = \left\langle i_s \right\rangle_{T_s}$$

$$\rightarrow \quad i_g = \left(\frac{v_m}{R_s V}\right) v_g$$

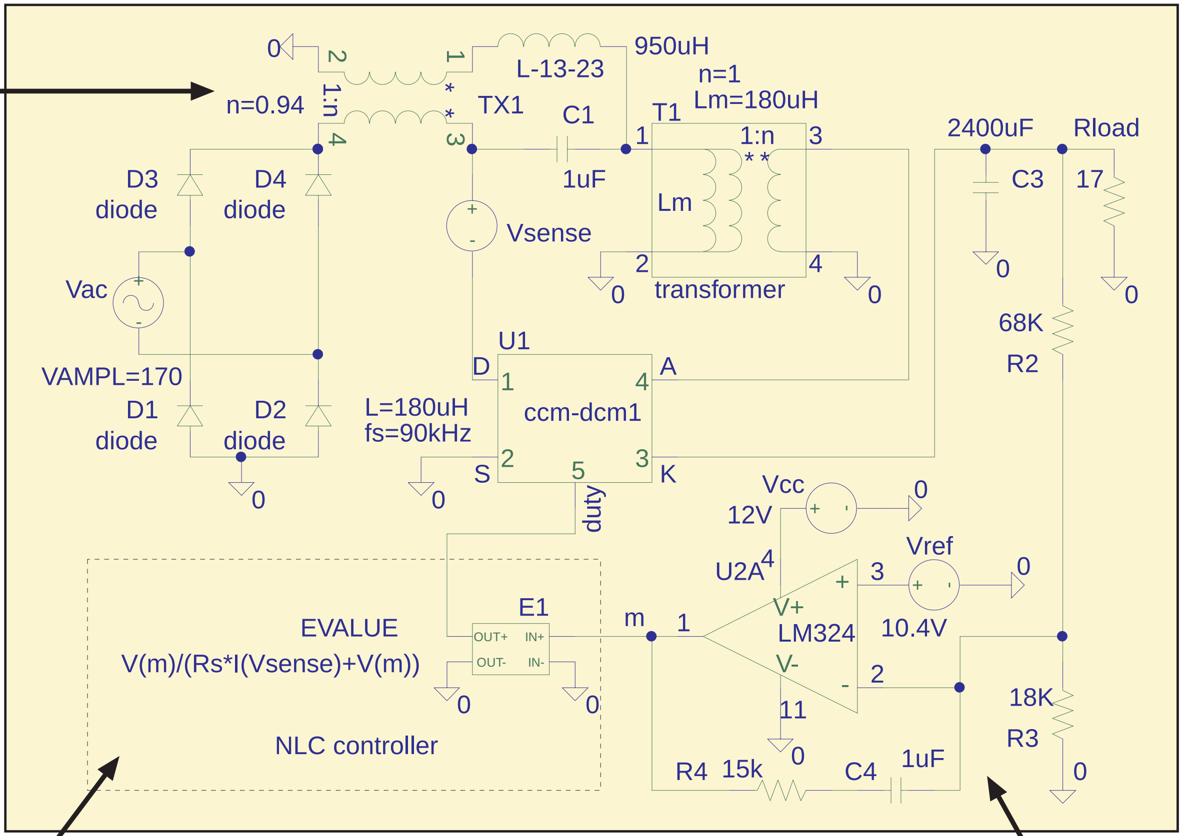
→ Ideal current shaping

$$d = \frac{v_m}{R_s \left\langle i_s \right\rangle_{T_s} + v_m}$$

NLC Controller Model

Sepic PFC with NLC Control: Simulation Model

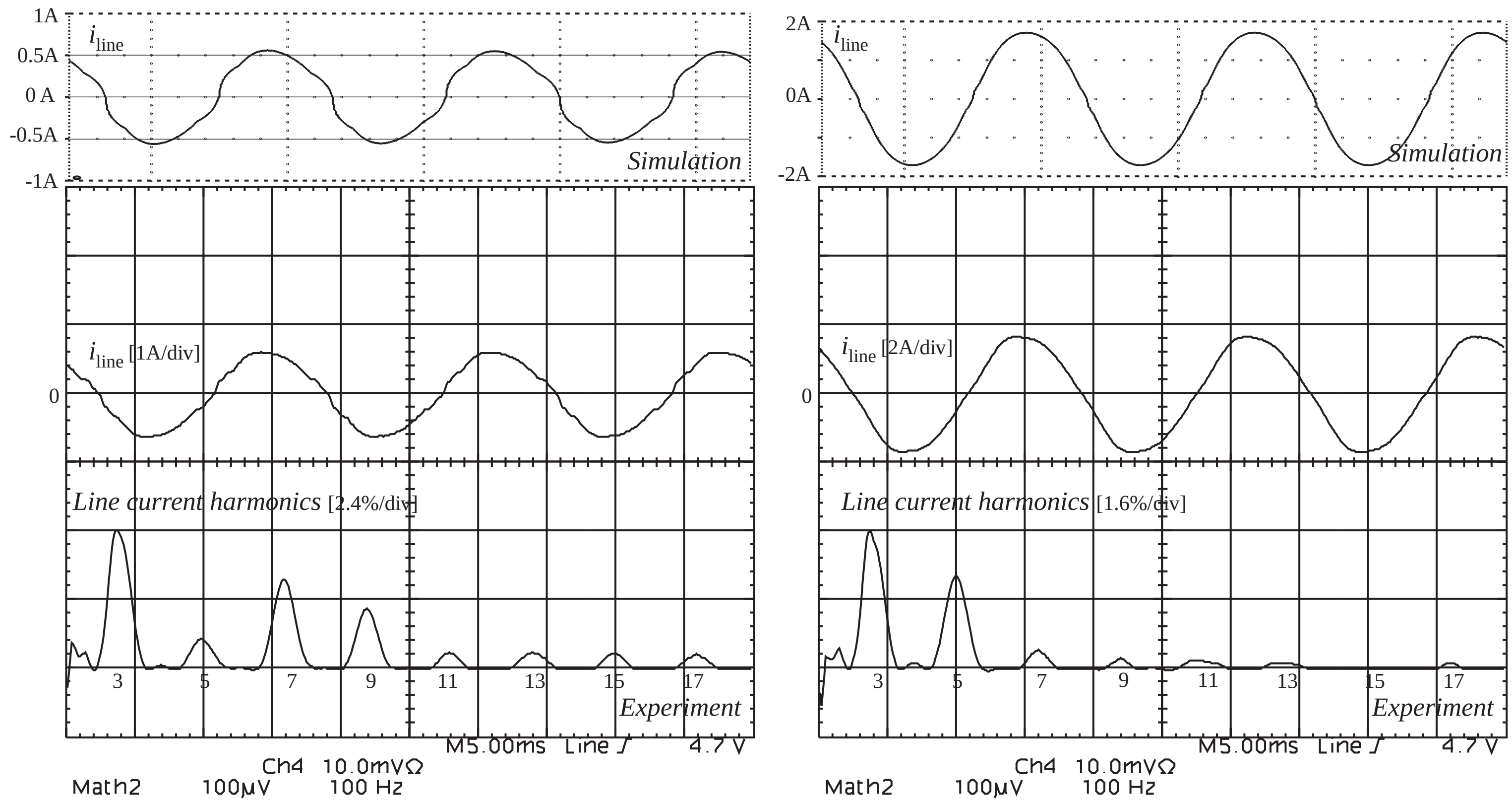
Coupled-inductor model



NLC controller model

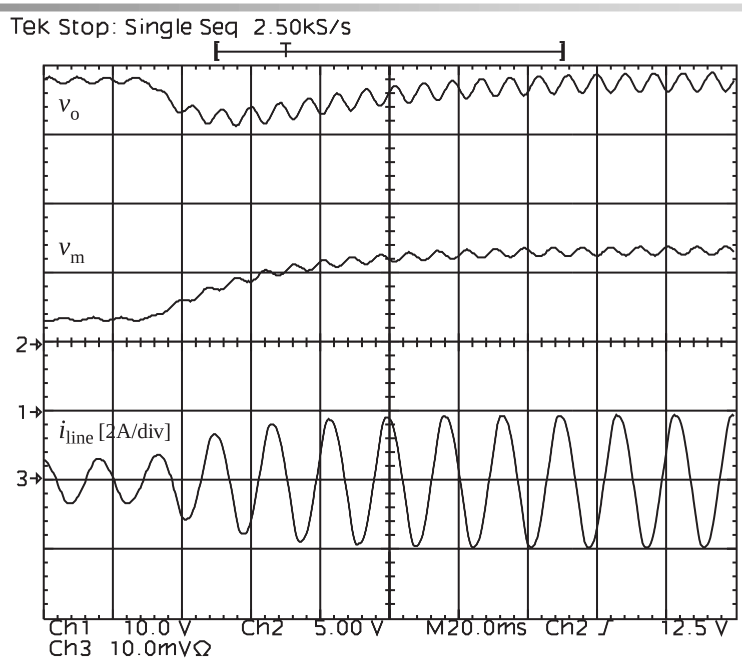
Closed-loop output voltage control

Sepic PFC with NLC Control
Experimental vs. Simulation Results

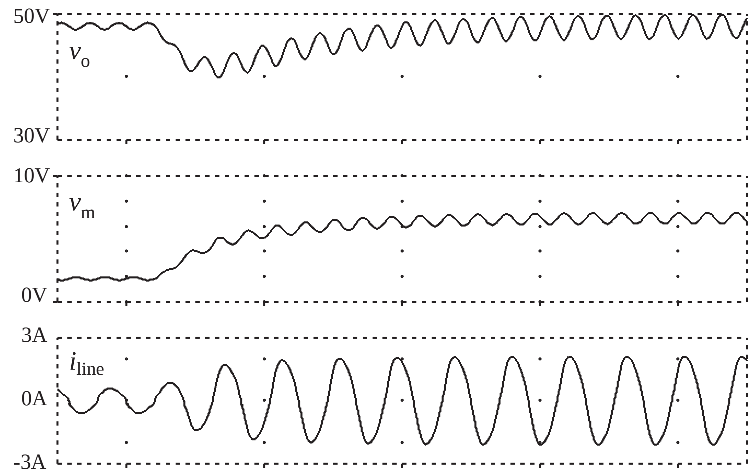


AC line current waveform and spectrum at 50W load (left) and 170W load (right)

Sepic PFC with NLC Control
Experimental vs. Simulation Results



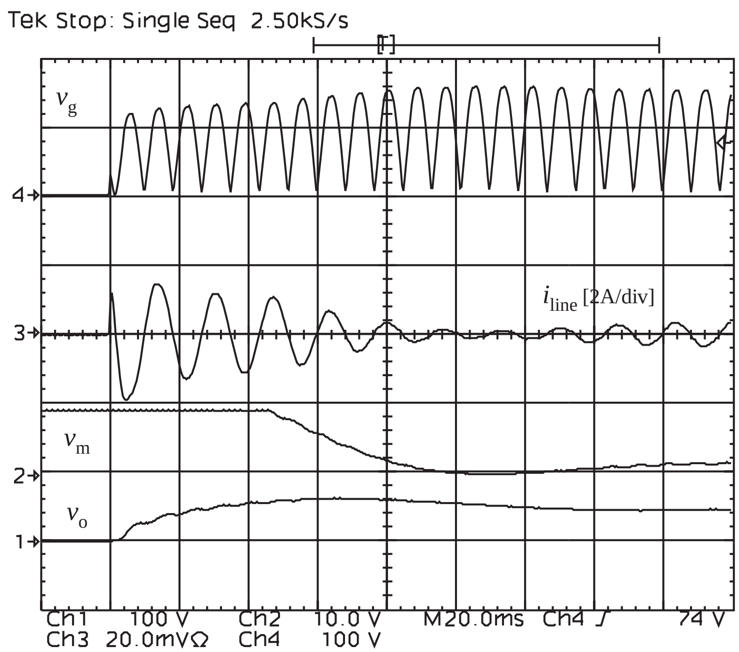
Experiment



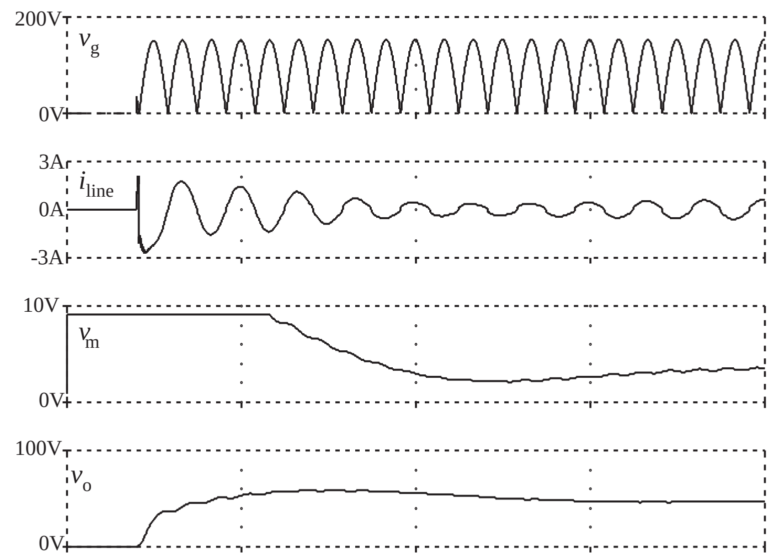
Simulation

50W to 125W load transient in the Sepic PFC

Sepic PFC with NLC Control
Experimental vs. Simulation Results



Experiment



Simulation

Start-up transient in the Sepic PFC at 50W load

6. Summary

- The averaged switch modeling approach: replace switch network with an equivalent circuit that correctly predicts the low-frequency components of the switch network terminal waveforms
- Seminar addressed:
  - PWM converters in continuous and discontinuous conduction modes
  - PWM converters with current-programmed mode (CPM) control
  - Single-phase low-harmonic rectifiers (power-factor correctors)
- In each case, the large-signal averaged switch model can be used:
  - to develop steady-state and (by linearization) small-signal circuit models suitable for analysis
  - to construct Spice-compatible model implementations suitable for DC, Transient and AC simulations
- A number of PSpice model implementation examples and converter application examples were presented

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